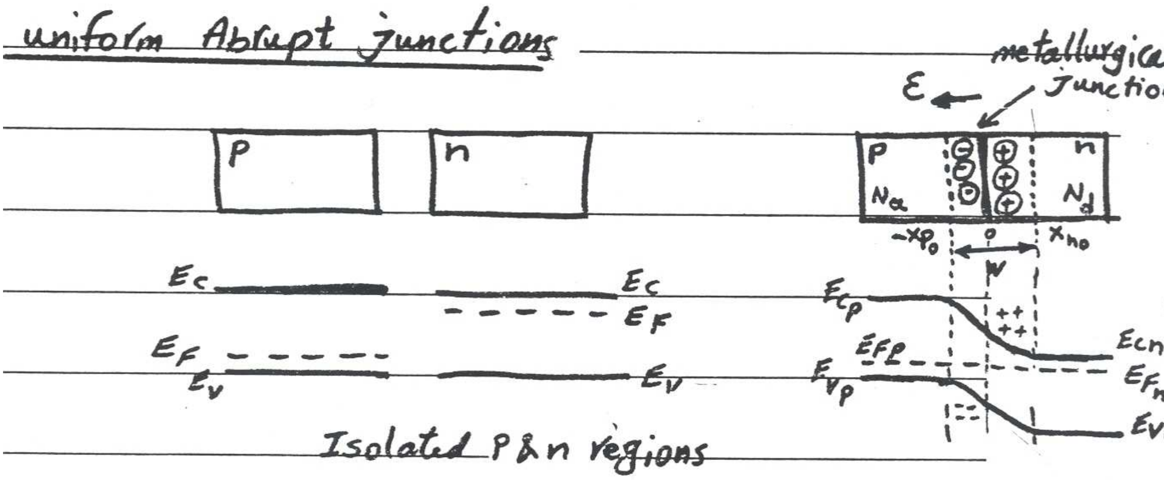
PART III
SEMICONDUCTOR DEVICES

Chapter 3- Semiconductor Diodes:
Physics of junction diodes which are essential components in almost all semiconductor devices and IC's, are discussed in this chapter.

3.1-PN Junctions:
A major component in junction diodes, BJT's, FET's and MOSFET's is the PN junction.

3.1 a- Uniform Abrupt junctions:

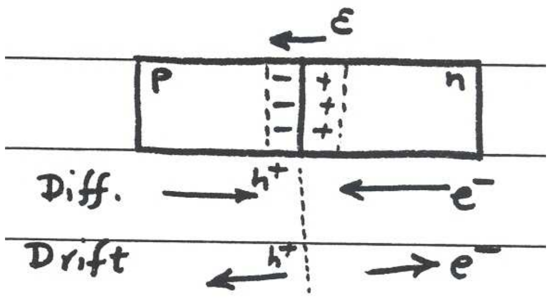


Note: $dE_f/dX = 0 \Rightarrow E_{fp}=E_{fn}$ at equilibrium.

Terminology: The transition region from p to n (of width w) is called the depletion region, since it is depleted of mobile charge. The immobile ionized impurity atoms (N_a^-, N_d^+) are left behind in this region, thus we may call it a space-charge region as well.

Observation 1: the immobile charges exposed in the transition region result in an E. Field from n to p.

Observation 2: The created E-Field introduces the balancing force to stop the diffusion process of e^{-}s moving from n to p and holes from p to n. This E-field also causes drift current for holes in the region to move toward the p-region and conduction e^{-}s in p to move into the n-region.



At equi. $J_p(drift)+J_p(diff)=0$ (3.1)

$J_n(drift)+J_n(diff)=0$ (3.2)

3.1 => $J_p(x) = q[\mu_p p(x)\varepsilon(x)-D_p dp/dx] = 0$

=> $\mu_p/D_p\ \varepsilon(x) = 1/p(x)(dp/dx)$ (3.3)

But $\varepsilon(x) = -dv/dx$; $D_p/\ \mu_p p = kt/q$

(3.3) => $-q/K_T(dv/dx) = 1/p(x)*(dp/dx)$

Integrating from $-X_{p0}$ to X_{no} yields

$-q/kt\int_{vp}^{vn} dv = \int_{pp}^{pn} dp/p$ => $-q/K_T(V_n-V_p) = \ln P_n/P_p$

Potential difference $V_n-V_p = 0$

Therefore, V_0 = kt/q ln P_p/P_n
(3.4)

For a uniform step junction : $P_p=N_a$, $n_n=N_d$ =>$P_n=n_i^2/n_n= n_i^2/ N_d$

Therefore, Kt/q (ln $N_a N_d/ n_i^2$) => built-in Potential or contact potential
(3.5)

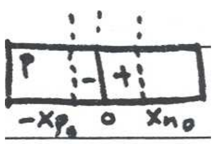
Alternate forms:

Eq. 3.4 => $P_p/p_n=e^{qvo/kt}$
(3.6a)

Using $P_p n_p=n_i^2=p_n n_n$ $n_n/n_p= e^{qvo/kt}$
(3.6b)

3.1b- Electric field and potential function calculations:

Gauss's Law: $d\varepsilon/dx =q/\varepsilon*(\rho_v)=q/ \varepsilon\ (p-n+N_d^+-N_a^-)$
(3.7)



In the space charge region:

TABLE 1

$-X_{po}<X<0$	$0<X<X_{n0}$
$\frac{d\varepsilon}{dX} = -\frac{q}{\varepsilon} N_a$	$\frac{d\varepsilon}{dX} = -\frac{q}{\varepsilon} N_d$
$\int_{-xp0}^{0} \frac{d\varepsilon}{dX} dx = \int_{-xp0}^{0} \frac{q}{\varepsilon} N_a dx$ = $\int_{0}^{Xno} \frac{q}{\varepsilon} N_d dx$	$\int_{0}^{Xno} \frac{d\varepsilon}{dX} dx =$
$\varepsilon(x) = -q\frac{Na}{\varepsilon}(X+XP_0)$ $X_{nd})$	$\varepsilon(x) = -q\frac{Nd}{\varepsilon}(X-$
At X=0 $\varepsilon_m = \frac{\mathbf{qNaXP0}}{\varepsilon}$	At X=0 $\varepsilon_m = \frac{\mathbf{qNdXno}}{\varepsilon}$

$\varepsilon = -\frac{dV}{dX}$ =>

$V(x) = \frac{qNa}{\varepsilon}(X^2/2 + XP_0 X) + K_b$ $\quad -X_{po}<X<0$

$V(x) = \frac{-qNd}{\varepsilon}(X^2/2 - X_n X) + K'_b$ $\quad 0<X<X_{n0}$

Let V=0 at $X=-XP_0$ =>

$K_b = qN_a/\varepsilon \ * \ Xp^2/2$

$K'_b = qN_d/\varepsilon \ * \ Xp^2/2 + V_o$

Check: X=0 ➔ $V(0) = k_b = -k'_b + V_o$

Abrupt and uniform pn junction

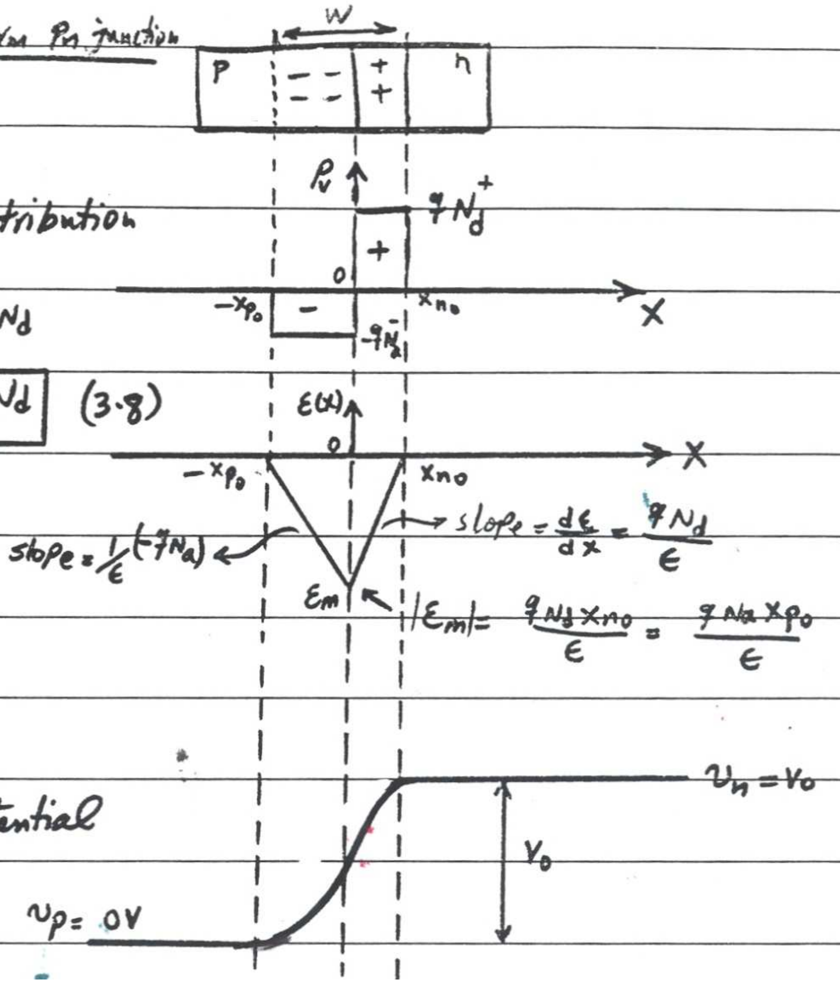
- charge distribution

$Q = qAx_{p_0}N_a = qAx_{n_0}N_d$

$\Rightarrow \boxed{x_{p_0}N_a = x_{n_0}N_d}$ (3.8)

- E-field

- Electrostatic Potential



3.1- Width of the depletion region (under equilibrium):

Vp=0

Vn=V(x) / X= Xn   qNdX2no /2ε + qNaX2po /2ε = Vo   (3.9)

$N_aXP_o = N_dXn_o$ (3.10)

2 Equations and 2 unknowns Xn_o and Xp_o but needs much work!

A better way:

⇨ Let $Xp_o+Xn_o=W \Rightarrow Xn_o=WN_a/(N_a+N_d)$ (3.11)

$\varepsilon= -dv/dx \Rightarrow v_o^{vo} = \int_{-xpo}^{xno} \varepsilon dx$ i.e. V is the neg. of area of under the curve E-Filed.

$V_0=-1/2\ (\varepsilon_m w) = 1/2\ (q/ \varepsilon * N_dX_{no}w) \Rightarrow V_o=1/2\ (q\ N_a\ N_dw^2/ \varepsilon(N_a+N_d)$

$$W=\left[2\ \varepsilon V_o/q\ (1/N_a+1/N_d)\right]^{½} = \left[2\ \varepsilon V_o/qN_B\right]^{1/2} \quad (3.12)$$

Where $1/N_B=(1/N_a+1/N_d)$

special case:

$p^+n= N_B=N_d$

$n^+p= N_B=N_a$

$$X_{po} = WN_d/(N_a+N_d) = \left\{ 2\,\varepsilon V_o/q \left[N_d/N_a(N_a+N_d) \right] \right\}^{1/2} \qquad (3.13a)$$

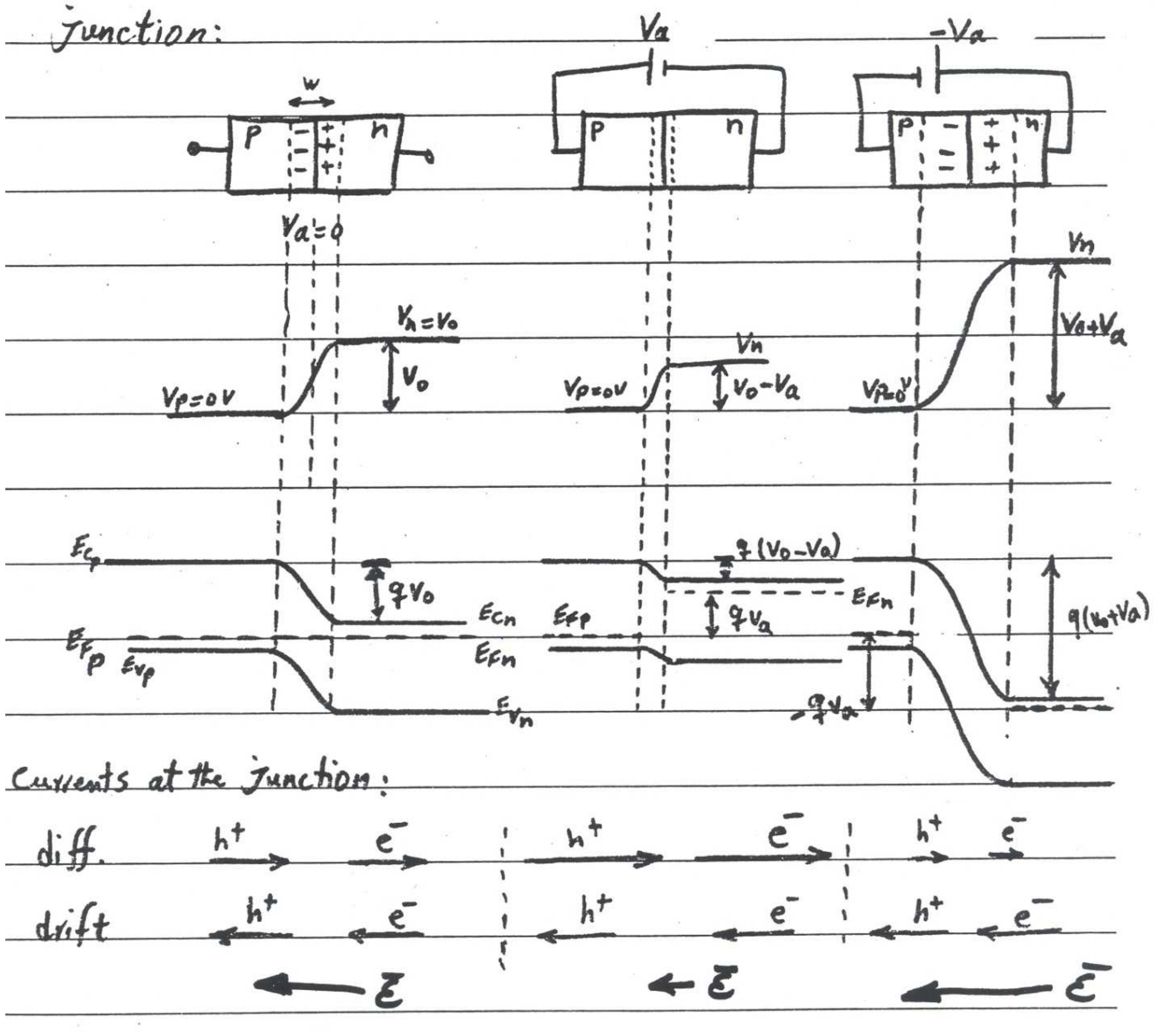
$$X_{no} = WN_a/(N_a+N_d) = \left\{ 2\,\varepsilon V_o/q \left[N_a/N_d(N_a+N_d) \right] \right\}^{1/2} \qquad (3.13b)$$

For a P^+n junction $N_a >> N_d$

$$\left\{ \begin{array}{l} V_0 = qN_d w^2/2 \\ \\ W=(2V_o/qN_a)^{1/2} \end{array} \right. \Rightarrow Xn_o \approx W \approx \left[2\,\varepsilon kt/q^2 N_d * \ln\ N_a N_d/n_i^2 \right]^{1/2} \qquad (3.13c)$$

3.2-Forward and reverse biased junctions:

Let us begin a qualitative discussion on the effects of applied bias and the important features of the junction.



Note: Potential energy barriers for e^-s and h^+s are directed oppositely. The barrier for e^- is apparent from energy band diagram, which is always drawn for e^- energies. For h^+, the potential energy barrier has the same shape as the electrostatic potential barrier multiplied by q.

Observation: The diffusion component of current is sensitive to the applied voltage (V_a), where as the drift component is relatively insensitive to the potential height variation due to a change in applied voltage.

Reason: e^- drift current is swept from p to n (due to stronger E) but rather on how many e^-s are swept down the barrier per second since there are very few minority e^-s in the p-side to participate (by wandering into the depletion region, or being generated within a diffusion length of the transition region which then can diffuse into this region).

3.2a- FWD Bias:

ANALYSIS:

Eq 3.6=> $P_p/P_n = e^{qvo/kt}$ (3.14a) equil. (zero bias)

$P(-X_p)/P(X_n) = e^{q(vo-va)/kt}$ (3.14b) with applied bias

Replacing for V_o => eq (3.14b) =>

V_a=KT/q*ln $P(X_n)/P(-X_p)$ + KT/q *ln N_aN_d/n^2_i (3.15)

Assumption of Low Injection:

This required that excess minority carrier concentrations vary significantly but changes in majority carrier concentrations are negligible i.e.

$P(X_n)>>P_o(X_n)$, $n(-X_p)>>n_o(-x_p)$ minority carrier.

$P(-X_p) \approx P_o(-X_p) = P_p$, $n(+X_n) \approx n_o(X_n) = n_n$ majority carrier

Note: $P(X_n) = P_o(X_n) + P'(X_n)$, $n(-x_p) = n_o(-x_p) + n'(-x_p)$

Where p' and n' are excessive minority carrier concentrations.

Therefore excess minority carrier concentrations are calculated as follows:

EQ (3.14b) => $P_p/P(x_n) = e^{qvo/kt}$ for low level injection

EQ (3.14a) => $P_p/P_n = e^{qvo/kt}$ (3.16)

Low Injection => $P(X_n)/P_n = e^{qvd/kt}$ (3.17)

Therefore, $V_a = kt/q * \ln P(X_n)/p_n$ where $P_n = P_o(X_n)$ (3.18)

Similarly Low Injection : $n(-X_p)/n_p = e^{qva/kt}$ (3.19)

Where $n_p = n_D(-x_p)$

Eqs (3.17) and (3.19) are called law of the junction obtained under low injection level.

Observation: Law of the junction states that excess minority carrier concentration is only caused by the applied voltage (V_a)

Excess minority carriers are:

$n'(-X_{po}) = n(-X_p) - n_p = n_p[e^{qvd/kt} - 1]$ (3.20a)

$p'(X_n) = P(X_n) - P_n = P_n[e^{qvd/kt} - 1]$ (3.20b)

Eq 3.20 describe minority carrier injection due to forward bias. These injected minority carriers diffuse deeper into neutral regions with a diffusion length L_p(or L_n). As the excess carriers diffuse, they recombine giving an exponential distribution. (See example 2 in CH.2)

Note: $L_p=(D_pT_p)^{1/2}$ (3.20c)

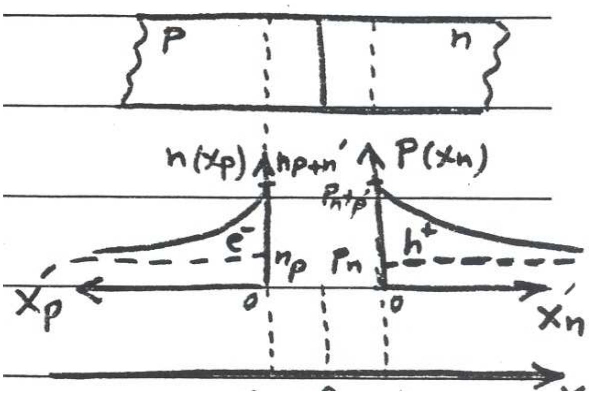
$L_n=(D_nT_n)^{1/2}$ (3.20d)

CASE I : Assume the neutral regions are indefinitely long.

Use continuity eq and eq (2.12):

$X=X_{no}$ and $-X_{po}$ are the edges of the transition region at equilibrium.

$X=X_{n,}, X_p$ are the edges at non-equilibrium.



2 new coordinate axes:

$0 \rightarrow X'_n$ | $X'_n=0 \Rightarrow X=X_n$

$O \rightarrow X_p'$ | $X'_p=0 \Rightarrow X=-X_p$

Excess Minority carrier diffusion:

Therefore, $p'(X'_n) = p'(X'_{no})e^{-Xn/Lp} = P_{no}(e^{qva/kT}-1)e^{-X'n/Lp}$ (3.20e)

Note 1: $p'(X'_n)/_{x'n=0} = P'(X_n)$

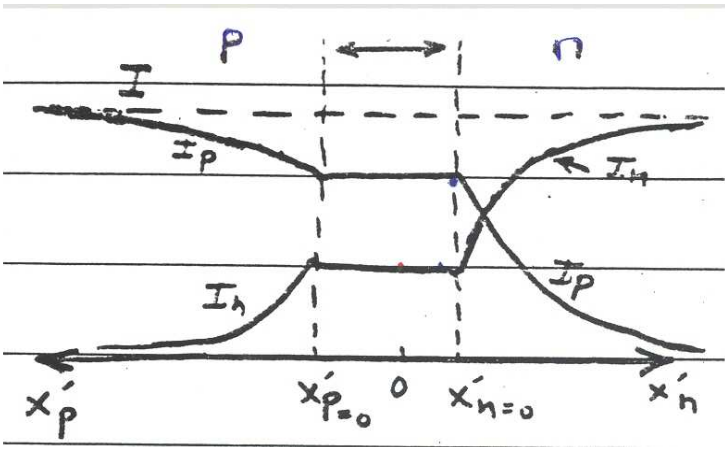
$n'(X'_p) = n'(X'_{po})e^{-Xp/Ln} = n_{po}(e^{qva/kT}-1)e^{-X'p/Ln}$

$I_p(X_n') = -q_A D_p \; dp'(X'_n)/d X'_n = qAD_p/L_p * p'(X_{no})e^{-X'n/Lp} = qAD_p/L_p * p'(X_n)$ (3.21a)

Similarly, $I_n(X'_p) = -qAD_n/L_n * n'(X'p)$ (3.21b)

$n'(X'_p)/_{x'p=0} = n'(X_p)$

Neglecting recombination in the depletion region then the total diode current I can be found by finding the current density at one point only since the total current is constant in this one-dimensional analysis.

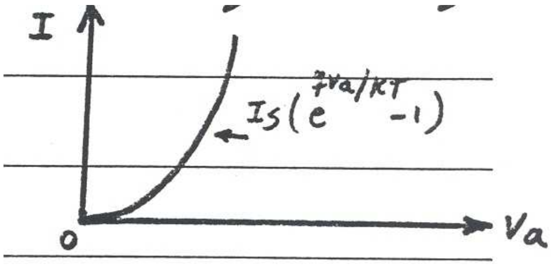


$I = I_p(x'_n=0) - I_n(x'_p=0) = qAD_p/Lp * p'(x'_n)/_{x'n=o} + qAD_n/Ln * n'(x'_p)/_{x'p=o}$

$I=qA(D_p/Lp * P_n + D_n/Ln * n_p)(e^{-qVa/kT}-1)$ (3.22)

$I = I_s (e^{qVa/kT}-1)$ Diode equation (3.23)

Where $I_s=q_A(D_p/L_p*P_{no} + D_n/L_n*n_{po})$ is called the reverse saturation current and represents the magnitude of the current when the applied voltage is large and negative.



Alternate approach to find I:

To find $I_p(X_n'=0)$, we see that this current is such that it should supply enough holes per sec to maintain the steady state exponential distribution $P'(X'_n)$ as the holes recombine.

$Q_p = qA\int_0^{\infty} P'(X'n)dX'n$ = $qAP'(X'_n=0)\int_0^{\infty} e^{-X'n/Lp}dx'n$-$qALpP'(x'_n=0)$

Therefore, $Q_p=qALpP'(x'_n=0)$ (3.24)

Average life time is T_p. Thus every T_p seconds, Q_p must be recombined.

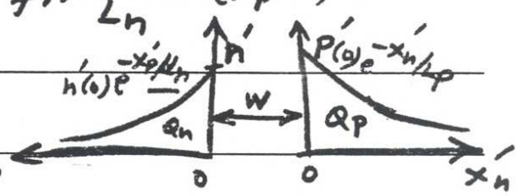
$I_p(X'_n=0) = Q_p/T_p = qALp/T_p p'(x'_n=0) = qAD_p/Lp * p'(x'_n=0) = qAD_p/Lp * p'(x'_n=0)$ (3.25)

Since ➔ $Lp = \sqrt{DpTp}$

Eq (3.25) is the same as eq (3.21a)

Similarly, $I_n(X'_p=0) = Q_n/T_n = qA\, D_n/Ln * n'(x'_p)=0$

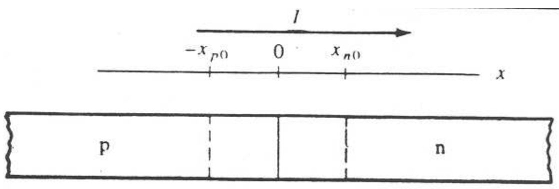
<u>Similarly</u> $I_n(x'_p=0)= \frac{Q_n}{T_n} = qA\frac{D_n}{L_n}n'(x'_p=0)$



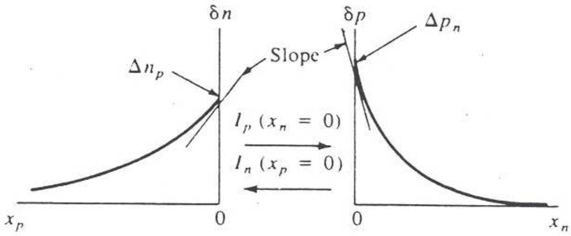
This provides the same result as eqn (3.23)

This provides the same result as eq (3.23).

Note: This method is called charge control approximation illustrating the important fact that the minority carriers injected into either side of a p-n junction diffuse into the neutral material and recombine with majority carriers.



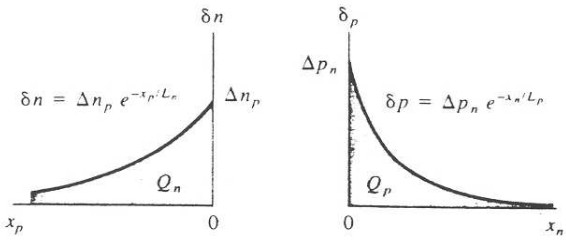
(a)



$$I_p(x_n=0) = -qAD_p \left.\frac{d\delta p}{dx_n}\right|_{x_n=0} = qA\frac{D_p}{L_p}\Delta p_n$$

$$I_n(x_p=0) = qAD_n \left.\frac{d\delta n}{dx_p}\right|_{x_p=0} = -qA\frac{D_n}{L_n}\Delta n_p$$

(b)



$$Q_n = -qA\int_0^{\infty} \delta n(x_p)\, dx_p \qquad Q_p = qA\int_0^{\infty} \delta p(x_n)\, dx_n$$

$$I_n(x_p=0) = \frac{Q_n}{\tau_n} = \frac{-qAL_n}{\tau_n}\Delta n_p \qquad I_p(x_n=0) = \frac{Q_p}{\tau_p} = \frac{qAL_p}{\tau_p}\Delta p_n$$

$$I = I_p(x_n=0) - I_n(x_p=0) = qA\left(\frac{D_p}{L_p}\Delta p_n + \frac{D_n}{L_n}\Delta n_p\right)$$

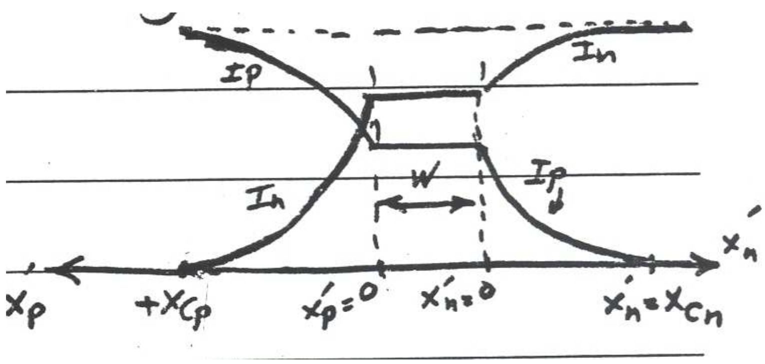
$$= qA\left(\frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n}\right)(e^{qV/kT} - 1)$$

Two methods for calculating junction current from the excess minority carrier distributions: (a) diffusion currents at the edges of the transition region; (b) charge in the distributions divided by the minority carrier lifetimes.

Case 2: The Neutral Regions are Finite:

Ohmic contacts are at X_{cn} and $-X_{cp}$

Let's analyze the current density due to holes in the n-side of the junction.



$P'(X'n)=Ae^{+x'n/Lp} + Be^{-x'n/Lp}$

B.C:

1) $P'(X'n=0) = P_{no}[e^{qVa/kT}-1]$

2) $P'(X'n=Xcn) =0$

Since for an ohmic contact, the excess carrier concentration is forced to zero.

Solving for an A and B results in:

$P'(X'n) = P_{no}\sinh[(X_{cn}-X'_n)/Lp][(e^{qVa/kT}-1)/\sinh[(X_{cn}/Lp]$ (3.26a)

Assuming the current due to minority carriers is entirely due to diffusion:

Then $J_p(X'n) = -qD_pdp/dx'_n = qD_pdp/dx'_n$

Therefore,

$J_p(x'_h) = D_p q P_{no} \cosh[(X_{cn}-x'_n)/Lp] * (e^{qVa/kT}-1)$ / $Lp \sinh[(X_{cn}/Lp]$ (3.26b)

Similarly,

$J_n(x'_p) = D_n q N_{po} \cosh[(X_{cp}-x'_p)/Ln] * (e^{qVa/kT}-1)$ / $Ln \sinh[(X_{cp}/Ln]$ (3.27)

At the edge of the transition region:

$J = J_n(X'_n=0) + j_p(X'_n=0)$ but $J_n(X'_n=0) = J_n(X'_p=0)$ no Recombination.

$J = J_p(X'_n=0) + j_n(X'_n=0) = J_s[e^{qVa/kT}-1]$ (3.28)

Where $J_s = \frac{qNpDn}{Lntanh\left(\frac{Xcp}{Ln}\right)} + qPnDp/(Lptanh(Xcn/Lp)$

$n_p = n_i^2/N_a$; $P_n = n_i^2/N_d$ and $I = JA$

From Eq (3.28) $I = I_s[e^{qVa/kT}-1]$ (3.29)

Where $I_s = qn_i^2 A \frac{Dn}{NaLntanh\left(\frac{Xcp}{Ln}\right)} + Dp/(NdLptanh(Xcn/Lp)$

Note 1: The excess minority and majority carrier concentrations are essentially same.

Note 2: The drift of minority carriers across the junction is commonly called "generation current" since its magnitude depends upon entirely on the rate of generation of EHP's.

Note 3: The diode under analysis is essentially a one dimensional device. And even though "I" was determined by considering only minority carrier diffusion, majority carrier currents are also present.

Note 4: Total current "I" is independent of position and is obtained by adding the minority carrier current and majority carrier current at each point.

3.2b- Quasi- Fermi levels:

Observation: Since the Fermi level is defined for equilibrium Condition, to describe junctions under non-equilibrium conditions we have to define two new quantities: E_{fn} and E_{fp}, the quasi Fermi levels for e^- and h^+ respectively.

Under Non-equilibrium:

$n = N_c e^{-(Ec-Efx)/kT}$ (3.30a)

$p = N_v e^{-(Efp-Ev)/kT}$ (3.30b)

Check: At equil. $E_{Fn}=E_{Fp}=E_F$ and give eqns (1.5) and (1.6)

The current density expression can be written in terms of the quasi Fermi levels.

$J_n=qn\mu_n E + qD_n dn/dx$ (3.31)

$dn/dx = \frac{n}{kT}\left(\frac{dEFn}{dx} - \frac{dEc}{dx}\right)$

$\varepsilon = -dv/dx = 1/q * \frac{dEc}{dx}$ (since V=-qEc+Constant)

$E_c = -qv+E_o$

Eq (3.31)➔ $J_n=q_n\mu_n \frac{dEFn}{dx}$ (3.32a)

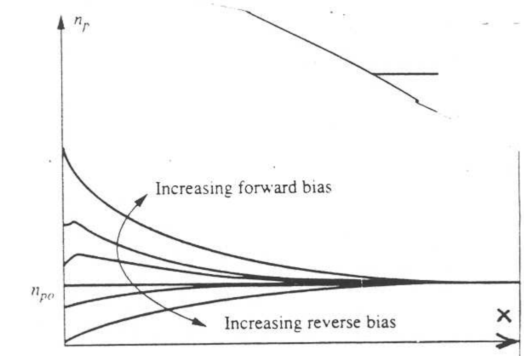
Similarly, $J_p=q_p\,\mu p \frac{dEFp}{dx}$ (3.32b)

It is clear from the results, to have a net current there must be a gradient of the quasi Fermi level.

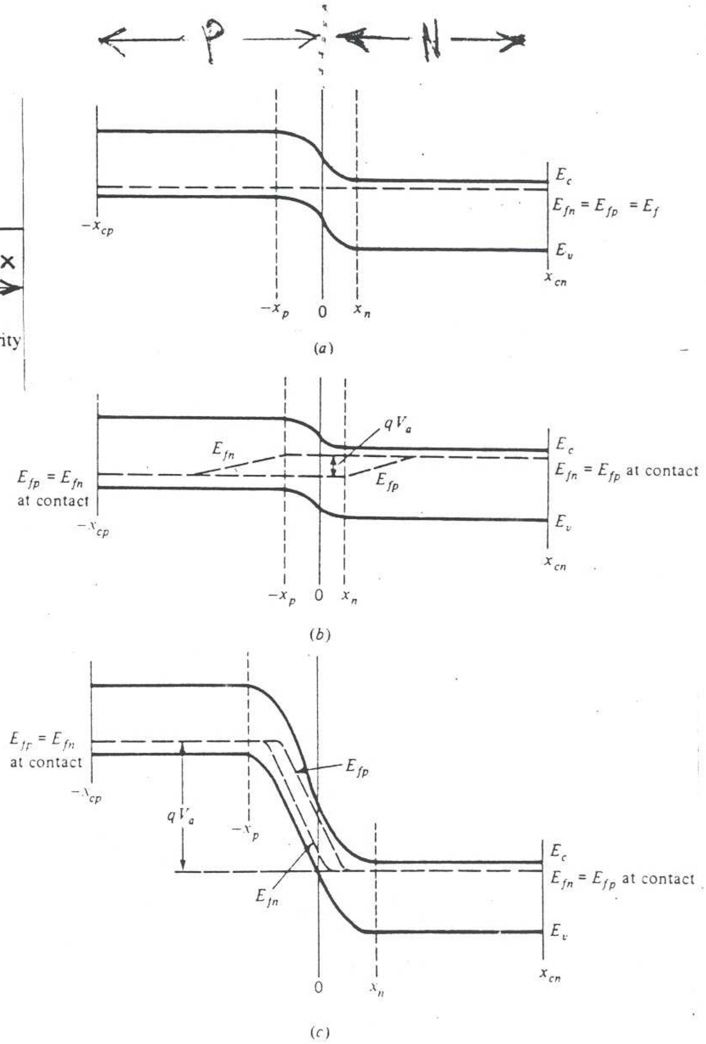
Check: If no current flows then $dE_F/dx = 0$ => E_F same everywhere.

Summary: Current at a p-n junction can be calculated in two ways.

1) From the slopes of the excess minority carrier distribution at the two edges of the transition region.
2) From the steady state charge in “p” and “n” distribution.



Change in the injected charge distribution when the polarity is Reversed



Quasi-Fermi levels in a *pn* junction for (*a*) equilibrium, (*b*) forward bias, and c) Reverse bias

3.2c-Reverse Bias:

This case is also obtained from eq (3.20)

For $+V_a<0$ & modV_a>> kT/q = .026v (T=300k)

$P'(X_n) = p(X_n) - p_n = p_n[e^{qVa/kT}-1] = -p_n$ (3.33a)

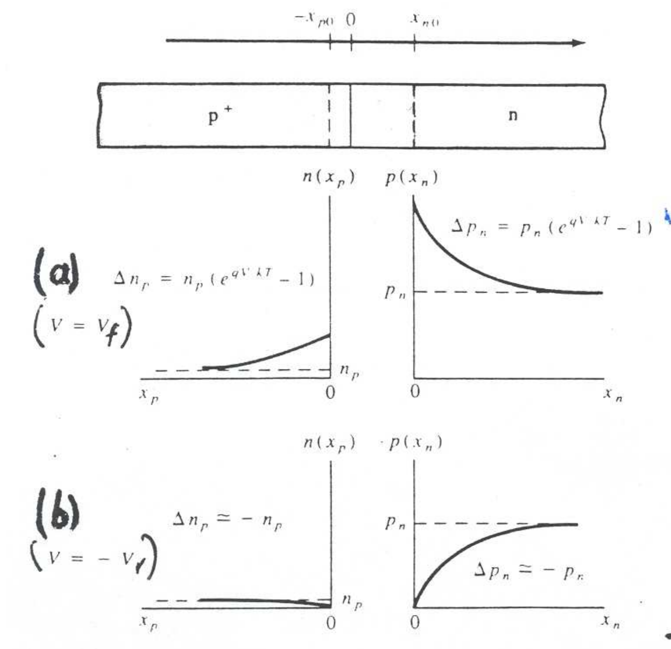
$n'(-X_p) = n_p[e^{qVa/kT}-1] = -n_p$ (3.33b)

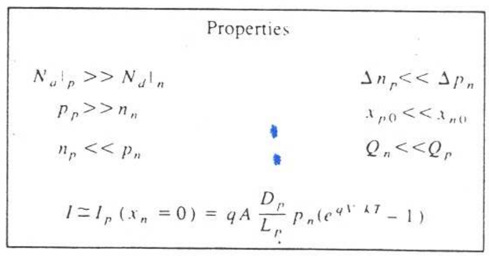
Observation:

Thus for V_a= a few tenths of a volt, the minority carrier concentration at each edge of the transition region becomes zero approximately extending a diffusion length beyond each edge into the neutral region as show below.

Terminology:

The reverse-biased depletion region of minority carrier is called a minority carrier "extraction" (analogous to injection of FWD bias)





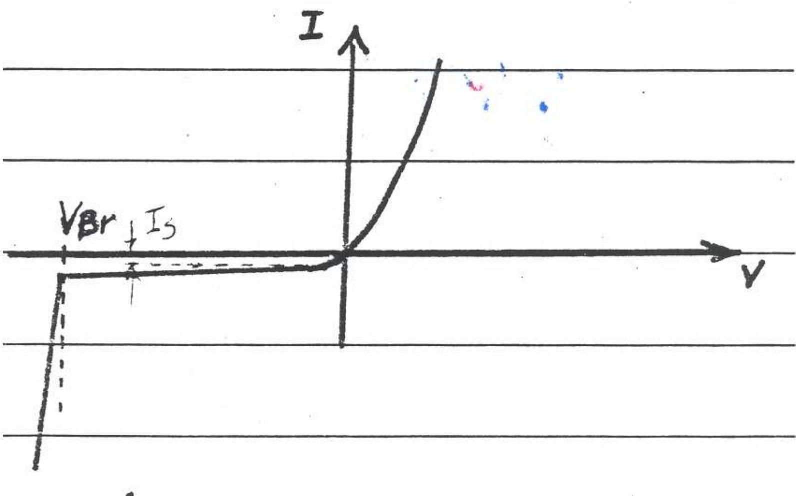
Properties of a p^+-n junction with
(a) forward bias and
(b) reverse bias.

Observation: For reverse bias: $+v_a<0$

$I = I_s(e^{qvo/kt}-1) \approx -I_s$ $\quad\quad$ $|V_a| >> Kt/q$

Which indicates that the reverse current goes to –Is and remains at that level independent of voltage.

In actuality: Diode show a gradual increase in reverse current with increasing reverse voltage until the breakdown voltage is reached. At this point, there is a significant increase in reverse current with almost no increase in voltage.



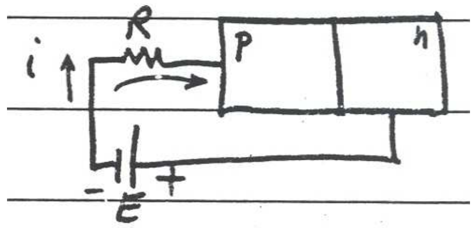
Observation: The increase in reverse current with increasing reverse voltage (below V_b) is due to gen. and recombination of EHP's in the transition region. (This was previously assumed nonexistent.) in most materials recombination and generation centers exist near the middle of the gap due to the impurities and effects in lattice, this causes the increase in reverse current with increasing voltage since the transmission region volume is also increasing causing a higher generation of EHP's.

3.2e-Breakdown:

Fact: There is nothing inherently destructive about the reverse breakdown provided the current is limited to within power consumption of the diode.

Ex:

$i_{max} \leq E\text{-}V_{Br}/R$ (3.34)

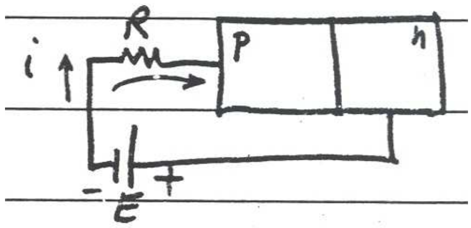


Breakdown can be due to two mechanisms:

A – Tunneling (or Zener Breakdown)

B – Avalanche Breakdown.

A- Zener Breakdown: Is operative at low voltages (V_r<5v). Tunneling is a process in which electrons penetrate into empty e^- states if the potential barrier is sufficiently narrow (<10nm). Required critical field for tunneling is about ≈10^6v/cm.



Basic requirements of Tunneling:

1- A large no. of e^- separated from a large no. of empty states.
2- A narrow barrier of finite height.

These lead to a sharp p-n junction and doping on both sides high => small w

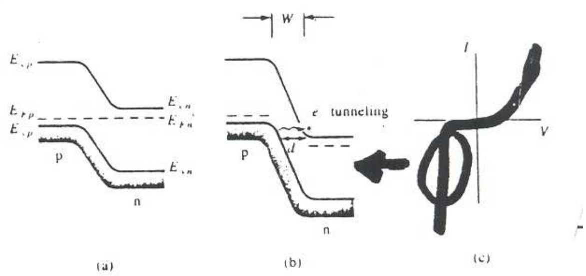


Figure 5-17 The Zener effect: (a) heavily doped junction at equilibrium; (b) reverse bias with electron tunneling from p to n; (c) I–V characteristic.

In the simple covalent bonding model, Zener effect can be thought of as field ionization of the host atoms at the junction, i.e. at critical e-filed e^-s in covalent bond may be torn from the bonds by this strong field and accelerated to the n-side of the junction.

B- Avalanche Breakdown:

The avalanche process occurs when carriers in the transition region are accelerated b y The E-filed to energies sufficient EHP's via collisions with bound e^-s. These secondary e^-s and h^+s(produced by these collisions) are also accelerated in turn by the field and thus produce tertiary EHP's.

Observation 1: Ionization probability proportional to e-Filed is proportional to V_{Br}.

Carrier multiplication is empirically found and is given by:

$M=[1-(-V_a/V_{Br})^m]^{-1}$ $2 \leq m \leq 6$ and $v_a<0$ (3.35)

For si 2<m<4 but on an average n≈3.

Note: For V_{Br}>8v, the avalanche mechanism is dominated one.

For $5 \leq V_{Br} \leq 8$, Breakdown is due to both tunneling and Avalanche process.

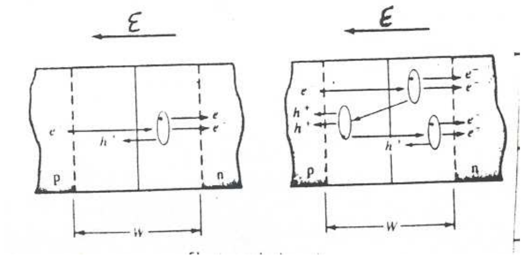
For a P^+N Junction, $\varepsilon_{max} = qN_dX_{no}/\varepsilon$, $X_{ho}=w \Rightarrow \varepsilon_{max} \propto 1/\sqrt{Nd} * Nd = \sqrt{Nd}$

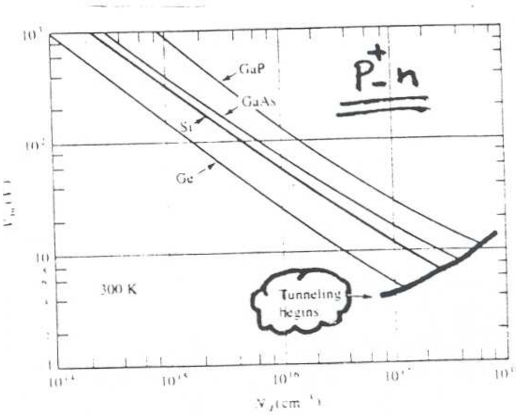
$V_{br}= \varepsilon_{max}\, w/2$ $w \propto 1/\sqrt{Nd}$

Observation 2: When E_g increases then V_{Br} increases. Since a higher field is needed to ionize.

When (N_a,N_d) increases then ε_{max} increases which implies V_{Br} decreases.

Figure below clearly shows this:





Variation of avalanche breaks down voltage vs. donor concentration

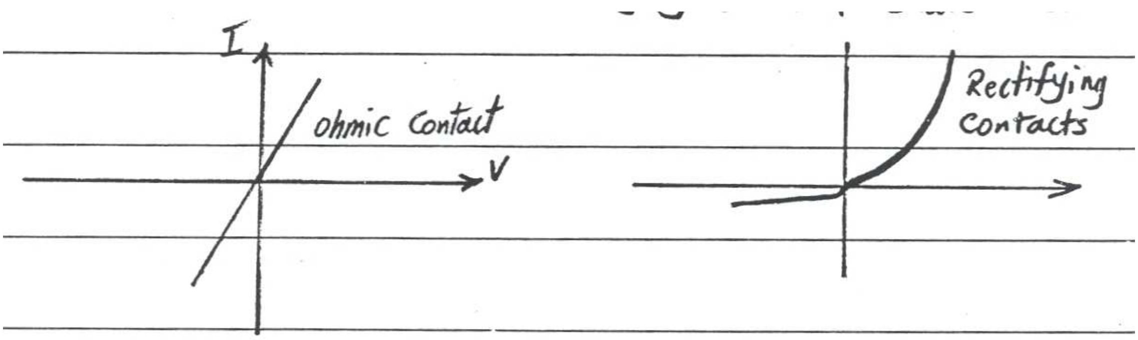
3.3-Metal semiconductor Junctions:

Theoretical Observations:

When a metal and a semi conductor are brought into contact, there is usually a redistribution of charge between the two materials before an equilibrium condition can exist.

Fact: This charge redistribution causes the band structure of the s/c to be disturbed in the vicinity of contacts.

Metal – s/c contacts:
1- Ohmic contacts: are bidirectional.
2- Rectifying contacts: Behave diode like.



Terminology:

Work Function: ($q\emptyset$): is defined as the energy required removing an e^- at the Fermi energy, from the material. (Note this is a general def. applying to s/c even if no e^- at E_F)

Electron Affinity: ($q\chi$): is defined as the energy required removing an e^- from an s/c when the e^- is at the bottom of the conduction band (E_c).

Solved Examples for Chapter 3

EX 13 B)

An n-type sample of Ge contains $N_4 = 10^{16}cm^{-3}$. A junction is formed by alloying within at 160°C. Assume that the acceptor concentration in the re-grown region equals the solid solubility at the alloying temperature.

(a) Calculate the fermi level positions at 300K in the p and n regions.
(b) Draw an equilibrium band diagram for the junction and determine the contact potential V_o from the diagram.
(c) Compare the results of part (b) with V_o as calculated from eq (3.5)

Solution:

From Appendix V, $N_a = 3\times 10^{18}cm^{-3}$.

(a) Assume That $P_p = N_a$, On the p side:

$$P_p = n_i e^{(Ei-Efp)kT}$$

Thus,

$$E_{ip}=E_{FP} = kTln\ P_p/n_i$$

$$= 0.0259ln\ (3\times 10^{18})/2.5\times 10^{13}$$

$$= 0.303eV$$

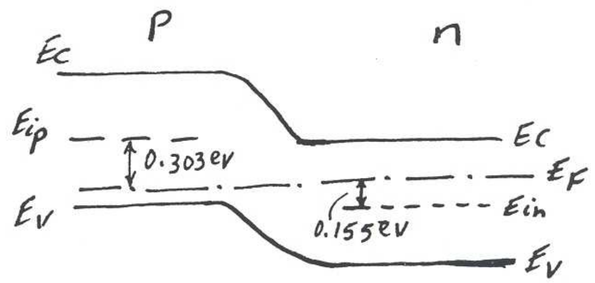
The resulting band diagram is shown

(b) $qV_o = E_{cp}\text{-}E_{cn} = E_{ip}\text{-}E_{in}$

$V_o = 0.303+0.155 = 0.458V.$

(c) $V_0=0.0259ln\ (3\times 10^{18}\times 10^{16})/6.25\times 10^{26}$

$= 0.0259ln(4.8\times 10^{7}) = 0.458V$



EX 13C)

Aluminum is alloyed into an n-type Si sample ($N_d = 10^{16}cm^{-3}$), forming an abrupt junction of circular cross section, with a diameter of 20mils. Assume that the acceptor concentration in the alloyed re-grown region is $N_a= 4\times 10^{18}cm^{-3}$. Calculate V_o,x_{no},x_{po},Q_+, and ε_0 for this junction at equilibrium (300k). Sketch ε(x) and charge density to scale.

Solution: From Eq (3.5),

$$V_o= kT/q \ln (N_aN_d/n_i^2)$$

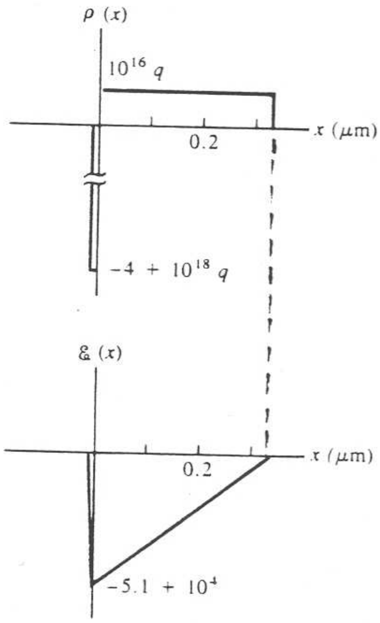
$$= 0.0259\ln (1078\times 10^{14}) = 0.85V$$

From $_{eq}$ (3.12),

$$W= \left[\frac{2\varepsilon Vo}{q}\left[\frac{1}{Na}+\frac{1}{Nd}\right]\right]^{1/2}$$

$$= \left[2(11.8\times 8.85\times 10^{-14})(0.85)\,(0.25)\times 10^{-18}+10^{-16})/(1.6\times 10^{-19})\right]^{1/2}$$

$$= 3034\times 10^{-5}cm = 0.334\mu m.$$



From eq (3.13),

X_{no} = 3.34× 10^{-5}/(1+0.0025) = 0.333μm.

X_{po} = 3.34× 10^{-5}/(1+400) = 8.3A.

Note: X_{no} = W.

A = πr^2 = π (2.54× 10^{-2})^2

= 2.03× 10^{-3}cm^2

Q_ = -Q_ = $qAX_{no}N_d$ = (1.6× 10^{-19})(2.03× 10^{-3})(3.33× 10^{-5})(10^{16})

= 1.08× 10^{-10}C

ε_o = $-qN_dx_{no}/\varepsilon$ = -(1.6× 10^{-19})(10^{16})(3.33× 10^{-5})/(11.8)(8.85× 10^{-14})

$= -5.1\times 10^{4}$V/cm.

Ex 13D)

A Si p-n junction has the following parameters:

$N_D = 10^{17}$ cm^{-3} , $N_A= 10^{15}$cm^{-3} , Area = 50µm$\times$ 50µm.

Find:

(a) The reverse saturation current
(b) The depletion region widths and the corresponding junction capacitances at bias voltages of V=-10V and +0.75V
(c) The diffusion capacitance at V= +0.75V (T=300k)

Solution: In order to calculate the reverse saturation current of the diode using Eq. 3.23, we need various parameters associated with Si. From the tables in chapter 5, we find

$D_n = 38\times 10^{-4}m^2s^{-1}$, T_n = 50µs.

$D_p = 13\times 10^{-4}m^2s^{-1}$, T_p= 10µs. and $n_i = 1.4\times 10^{16}m^{-3}$.

The diffusion lengths are then equal to

$L_n = (D_nT_n)^{1/2} = 4.36\times 10^{-4}m$ and $L_p = (D_pT_p)^{1/2} = 1.14\times 10^{-4}m$.

I_o then becomes

$I_o = (2.5\times 10^{-9}m^2)(1.6\times 10^{-19}C)(1.4\times 10^{16}m^{-3})^2$

$\times \left[38\times 10^{-4}/(4.36\times 10^{-4})(10^{21}) + 13\times 10^{-4}/(1.14\times 10^{-4})(10^{23}) \right]$

$I_o = 6.92\times 10^{-16}$A.

(b) Before we can calculate the depletion width, we need the diode potential V_D. from Eq (3.18)

$$V_D = (0.026eV) \times \left[\ln\ 10^{21} \times 10^{23} / (1.4 \times 10^{16}) \right]$$

$$V_D = 0.701eV$$

From the reverse bias of -10.0V, the depletion layer W_d (from Eq 3.12) is

$$W_d = \left[(2 \times 11.8 \times 8.854 \times 10^{-12})(10.0+0.701)/(1.6 \times 10^{-19})(10^{21}) \right]^{1/2}$$

$$W_d = 3.74 \times 10^{-6}m$$

The corresponding junction capacitance is

$$C_j = (11.8 \times 8.854 \times 10^{-12})(2.5 \times 10^{-19})/(3.74 \times 10^{-16})$$

$$C_j = 6.98 \times 10^{-14}F.$$

Note: For v=+0.75, the v is greater than V_D and W_d becomes imaginary. This indicates that the low-level injection approximation does not hold any more, and it may be that the depletion layer disappears completely.

c) For the diffusion capacitance, we need the current through the diode at the operating bias potential. This is equal to

$$i = 6.92 \times 10^{-16}(\exp^{(0.75/0.026)} - 1)A$$

$$i = 2.33 \times 10^{-3}A = 2.33mA$$

The diffusion capacitance is then equal to

$$C_d = (50 \times 10^{-6}s)(2.33 \times 10^{-3}A)(0.026eV)$$

$$C_d = 3.03 \times 10^{-9}F = 3.03nF.$$

EX 14)

Determine the change in barrier height of a p^+-n junction diode at 300˚K when the doping on the n-side is changed by a factor of 1000, if the doping on the p-side remains unchanged

Solution

From Eq . (3.5)

$$(\Delta V_o)_1 = (KT/q)Tn\ [(N_a)_1(N_d)_1/n^2]$$

and

$$(\Delta V_o) = (KT/q)Tn\ [(N_a)_2(N_d)_2/n^2],$$

Where the subscript 1 refers to the lightly doped case and 2 to the heavily doped case, Substracting the first equation from the second gives

$(\Delta V_o)_2 - (\Delta V_o)_1 = (kT/q)\ In\ [N_a)_2(N_d)_2/(N_a)1(N_d)_1] = (0.026\ V)\ In(1000) =$ <u>0.18 V.</u>

Hence a 1000-fold change in doping alters the barrier height by only 180 mV.

EX 15 **CH. 3**

Determine the space-charge width of the n-region of an abrupt silicon p^--junction in thermal equilibrium at 300°K if the doping on the n-side is 1.0×10^{14} donors/cm^3 and that on the p-side is $5.0 \times 10^{10}/cm^3$. The relative dielectric constant of silicon is 12.

Solution

From Eq. (3.8)

$$l_n/l_p = N_a/N_d = (5.0 \times 10^{10})/10^{14} = 5.0 \times 10^5$$

Therefore the space-charge width in the p-region is negligibly narrow compared to that in the n-region. Hence using Eq. (3.13c) we have

$$l_n = \frac{2\varepsilon o\varepsilon s KT}{q^2 N_d}\left[\quad\frac{N_a N_d}{n_i^2}\right]^{½}$$

$$= \left[\left((0.026\ V)\ \ln\frac{(5.0\ X\ 10^{10})(1.0\ X\ 10^{14})}{(1.5\ X\ 10^{10})^2}\right)\right.$$

$$\left. x\quad \frac{2\ (8.85\ \ X\ \ 10^{-14}\ F/cm)12}{(1.6\ X\ 10^{-19}\ C)(10^{14}\ cm^{-3})}\right]^{1/2}$$

$$= [0.80(1.33\ X\ 10^4)]^{1/2} = \underline{3.3\ X\ 10^{-4}\ cm}$$

Ex 16

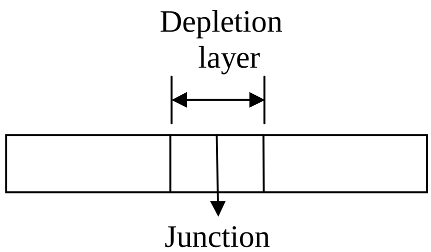
CH. 3

Assume the silicon p^-- n junction diode design as in Example ***15*** show that the balance of drift and diffusion currents in the space-charge region. Which is absolute under equilibrium conditions is not much disturbed by the application of a normal forward bias voltage. Say 0.60 V. at 300°K.

Solution

First let us roughly approximate the hole diffusion current in the space-charge region (which is equal and opposite to the hole drift current) under equilibrium conditions.

$$(J_{diff})_p = -\ qD_p \frac{dp}{dx}$$



From Example ***15*** the density of holes at the left-hand edge of the space-charge region is 5.0 X 10^{19} cm^{-3}. Since the *pn* product in equilibrium at 300°K is
$(1.5 \times 10^{10})^2$ cm^{-6}, the hole density at the right-hand edge of the space-charge region is $(1.5 \times 10^{10})^2/10^{-14}$ cm^{-3} or 2.25 X 10^6 holes/cm^3. Since the depletion-layer width from Example ***15*** is 3.3 X 10^{-4} cm,

$$-\left(\frac{dp}{dx}\right) \simeq \frac{5.0 \times 10^{19-} - 2.25 \times 10^{6}}{3.3 \times 10^{-4}} \quad {}^{-4}$$

$$= 1.5 \times 10^{23} \text{ cm}^{-4}$$

and

$$(J_{diff})_p \simeq (1.6 \times 10^{-19} \text{ C})(1.25 \times 10^{1} \text{ cm}^2/\text{sec})(1.5 \times 10^{23} \text{ cm}^{-4})$$

$$= \underline{3.0 \times 10^{3} \text{ A/cm}^2.}$$

So $(J_{diff})_p \simeq \underline{3.0 \times 10^{3} \text{ A/cm}^2.}$

Let us now calculate the hole current at the right-hand edge of the depletion layer, assuming in advance that the equilibrium drift and diffusion current balance is little upset by the application of a forward bias of 0.60 V. Using Eq. (3.21) and assuming that
L_p = 1.0 X 10^{-2} cm, we have

$$J_p = \frac{qD_pP_{no}}{L_p} \; (e^{qv/KT}-1)$$

$$= \frac{(1.6 \times 10^{-19} \text{ C})(1.25 \times 10^{1} \text{cm}^2/\text{sec})(2.25 \times 10^{6} \text{ cm}^{-3})}{1.0 \times 10^{-2} \text{ cm}}$$

$$\times \; (e^{0.60/0.026} - 1)$$
$$= \underline{4.7 \text{ A/cm}^2}$$

This represents only $4.7/(3.0 \times 10^5) \times 100$ or (1.6×10^{-3})%, or a minute upset of balance of equilibrium drift and diffusion currents, and thus the proposition is proved.

EX 17

CH. 3

Determine the thickness in micrometers of the space-charge region of a p'-n' junction, reverse biased by 6.0 V, near the onset of Zener break-down. Also find the peak field in the space-charge region under this condition. The p'-region is doped with $5.0 \text{ X } 10^{19}$ acceptor/cm^3 and the n'-region contains $2.0 \text{ X } 10^{IR}$ donor impurities/cm^3.

Solution

The calculation is identical to that carried out in Example ***15*** with the exception that an external voltage is applied in addition to the built-in voltage. Calculation of the built-in voltage by Eq. (3.5) yields $\Delta V_n = 1.06$ V. The space-charge width is then given by Eq. (3.13c) as

$$l_n = \left((\Delta V_o + |V|) \; 2\underline{\varepsilon o \varepsilon s} \right)$$

$$= \left((1.06 + 6.0) \quad \frac{2\,(8.85 \text{ X } 10^{-14})12}{\text{[illegible]}} \right)^{1/2} \text{cm}$$

$$= 6.9 \text{ X } 10^{-6} \text{ cm or } \underline{0.069\ \mu\text{m.}}$$

Equation (6.16) gives the peak electric field as

$$\left| \varepsilon_{max} \right| = \frac{2\,(\Delta V_n + |V|)}{l_n} = \frac{2\,(1.06 + 6.0 \text{ V})}{6.9 \text{ X } 10^{-6} \text{ cm}}$$

$$= \underline{2.1 \text{ X } 10^{6} \text{ V/cm.}}$$

EX 18

CH. 3 The leakage current I_o of a 2.0-cm^2 silicon p´ -junction solar celll at 300°K is 0.05 nA. The short-circuit current of this device exposed to noonday sun is 20 mA and the electron-hole pair generation rate in the silicon is then 3.0 X 10^{18}/cm^1-sec.

(a) What is the lifetime of minority holes in the n-region of this device?
(*Hint*: Assume that the electron lifetime in the p-region is very small because
of the high impurity level there)
(b) What resistance value must be connected across the cell in order to ensure that
10 mA of load current is delivered to this load?
(c) Calculate the power delivered to the load of (b).

Solution
(a)

$$I = I_0\left(e^{qV/kT} - 1\right) - \overbrace{(qAL_p g + qAL_n g)}^{I_L}$$

(generation rate)

$I_{sc} = I_L = qAG(L_p + L_n)$.

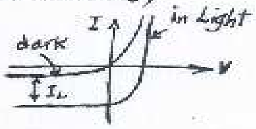
so

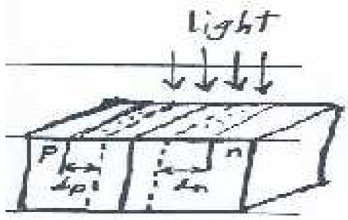
$L_p = I_{sc}/qAG$,

where it is assumed that $L_n \ll L_p$.
Hence

$$L_p = \frac{20 \times 10^{-3}\ A}{(1.6 \times 10^{-19}\ C)(2.0\ cm^2)(3.0 \times 10^{18}\ cm^{-3})}$$

$= 2.1 \times 10^{-2}$ cm.





only EHPs within L_n or L_p can diffuse to depletion region and be swept & contribute to I

and

$$T_p = \frac{L^2_p}{D_p} = \frac{(2.1 \times 10^2)^2\ cm^2}{12.5\ cm^2/sec}$$

$= 35 \times 10^{-6}$ sec or 35 μsec.

(b) Equation **(18a)** gives

$$I = I_o(e^{qV/KT} - 1) - I_L$$

Solving for *V* gives

$$V = \left(\frac{kT}{q}\right) \ln \left(\frac{I_L + I}{I_n} + 1\right)$$

$$= (0.026 \text{ V}) \ln \left[\frac{(20 \text{ X } 10^{-1}) + (-10 \text{ X } 10^{-3} \text{ A})}{.05\text{x}10^{-9}} + 1\right]$$

$$= 0.50 \text{ V};$$

R = V/I = 0.50/10 X 10^{-3} = <u>50 Ω</u>

(c) Power = *IV*
= (10 X 10^{-1} A) (0.50 V) = <u>5.0 X 10^{-1} W.</u>

EX 19

CH. 3

The capacitance of an abrupt. Long p^-- n junction diode 1.0 X 10^{-4} cm in area measured at – 1.0 V reverse bias is 5.0 pF. The built-in voltage ΔV_o of this device is 0.90 V. When the diode is forward biased with 0.50 V. a current of 10 mA flows. The n-region minority hole lifetime is known to be 1.0 µsec at 300°K.

(a) Calculate the depletion-layer capacitance of this junction at 0.50 V forward bias.

(b) Calculate the diffusion capacitance of the diode operating as in (a) at 300°K

Solution

(a) The depletion-layer capacitance of a p-n junction is given by Eq. (3.37) Hence in the forward direction at 0.50 V, the depletion-layer capacitance is

$$C_j = (5.0\ \text{pF})\ [(-\ 1.0 - 0.90)/(0.50 - 0.90)]^{1/2}$$

$$= \underline{11\ \text{pF}}$$

(b) From Eq. (3.41)

$$C_D = \frac{I\tau_p}{kT/q} = \frac{(10 \text{ X } 10^{-3}\ \text{A})\ (1.0 \text{ X } 10^{-6}\ \text{sec})}{0.026\ \text{V}}$$

$$= 38 \text{ X } 10^{-8}\ \text{F}$$

$$= \underline{380.000\ \text{pF}}$$

This example illustrates that the diffusion capacitance of a forward-baised junction diode can be four of more orders of magnitude greater that the depletion lay capacitance of a p-n junction. Then the p-n junction minority carrier charging time depends on the product of the junction dynamic resistance and the capacitance. This product for a long diode is essentially given by

$$r_{jct}C_D = \left(\frac{dv}{di}\right)_{jct}(C_D) = \frac{kT}{qI}\,\frac{qI\tau_p}{kT} = \tau_p$$

Minority carrier charge distribution in the n- region of a long p-n junction diode before and after the application of an incremental voltage dv. The cross-hatched area represents the additional role charge stored in the n-region as a result of the additional applied voltage dv.