

# Home work problems

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- \*9. Derive Eq. 17. (Hint: In the valence band, the probability of occupancy of a state by a hole is  $[1 - F(E)]$ .)
10. At room temperature (300 K) the effective density of states in the valence band is  $2.66 \times 10^{19} \text{ cm}^{-3}$  for silicon and  $7 \times 10^{18} \text{ cm}^{-3}$  for gallium arsenide. Find the corresponding effective masses of holes. Compare these masses with the free-electron mass.
11. Calculate the location of  $E_i$  in silicon at liquid nitrogen temperature (77 K), at room temperature (300 K), and at  $100^\circ\text{C}$  (let  $m_p = 1.0 m_0$  and  $m_n = 0.19 m_0$ ). Is it reasonable to assume that  $E_i$  is in the center of the forbidden gap?
12. Find the kinetic energy of electrons in the conduction band of a nondegenerate  $n$ -type semiconductor at 300 K.
13. (a) For a free electron with a velocity of  $10^7 \text{ cm/s}$ , what is its de Broglie wavelength.  
(b) In GaAs, the effective mass of electrons in the conduction band is  $0.063 m_0$ . If they have the same velocity, find the corresponding de Broglie wavelength.
14. The intrinsic temperature of a semiconductor is the temperatures at which the intrinsic carrier concentration equals the impurity concentration. Find the intrinsic temperature for a silicon sample doped with  $10^{15}$  phosphorus atoms/ $\text{cm}^3$ .

### FOR SECTION 2.7 DONORS AND ACCEPTORS

- 15. A silicon sample at  $T = 300 \text{ K}$  contains an acceptor impurity concentration of  $N_A = 10^{16} \text{ cm}^{-3}$ . Determine the concentration of donor impurity atoms that must be added so that the silicon is  $n$ -type and the Fermi energy is 0.20 eV below the conduction band edge.
16. Draw a simple flat energy band diagram for silicon doped with  $10^{16}$  arsenic atoms/ $\text{cm}^3$  at 77 K, 300 K, and 600 K. Show the Fermi level and use the intrinsic Fermi level as the energy reference.
- \* 17. Find the electron and hole concentrations and Fermi level in silicon at 300 K (a) for  $1 \times 10^{15}$  boron atoms/ $\text{cm}^3$  and (b) for  $3 \times 10^{16}$  boron atoms/ $\text{cm}^3$  and  $2.9 \times 10^{16}$  arsenic atoms/ $\text{cm}^3$ .
- 18. A Si sample is doped with  $10^{17} \text{ As atoms/cm}^3$ . What is the equilibrium hole concentration  $p_0$  at 300 K? Where is  $E_F$  relative to  $E_i$ ?
19. Calculate the Fermi level of silicon doped with  $10^{15}$ ,  $10^{17}$ , and  $10^{19}$  phosphorus atoms/ $\text{cm}^3$  at room temperature, assuming complete ionization. From the calculated Fermi level, check if the assumption of complete ionization is justified for each doping. Assume that the ionized donors is given by  $n = N_D [1 - F(E_D)] = \frac{N_D}{1 + e^{(E_F - E_D)/kT}}$ .
20. For an  $n$ -type silicon sample with  $10^{16} \text{ cm}^{-3}$  phosphorous donor impurities and a donor level at  $E_D = 0.045 \text{ eV}$ , find the ratio of the neutral donor density to the ionized donor density at 77 K where the Fermi level is 0.0459 below the bottom of the conduction band. The expression for ionized donors is given in Prob. 19.

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As the electric field becomes higher, the drift velocity departs from its linear relationship with the applied field and approaches a saturation velocity. This effect is particularly important in the study of short-channel field-effect transistors discussed in Chapter 6. When the field exceeds a certain value, the carriers gain enough kinetic energy to generate electron-hole pair by colliding with the lattice and breaking a bond. This effect is particularly important in the study of  $p$ - $n$  junctions. The high field accelerates these new electron-hole pairs, which collide with the lattice to create more electron-hole pairs. As this process, called impact ionization or the avalanche process, continues, the  $p$ - $n$  junction breaks down and conducts a large current. The junction breakdown is discussed in Chapter 4.

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## PROBLEMS (\* INDICATES DIFFICULT PROBLEMS)

### FOR SECTION 3.1 CARRIER DRIFT

1. Find the resistivities of intrinsic Si and intrinsic GaAs at 300 K.
2. Assume that the mobility of electrons in silicon at  $T = 300$  K is  $\mu_n = 1300$  cm<sup>2</sup>/V-s. Also assume that the mobility is mainly limited by lattice scattering. Determine the electron mobility at (a)  $T = 200$  K and (b)  $T = 400$  K.
3. Two scattering mechanisms exist in a semiconductor. If only the first mechanism is present, the mobility will be 250 cm<sup>2</sup>/V-s. If only the second mechanism is present, the mobility will be 500 cm<sup>2</sup>/V-s. Determine the mobility when both scattering mechanisms exist at the same time.
- 4. Find the electron and hole concentrations, mobilities, and resistivities of silicon samples at 300 K, for each of the following impurity concentrations: (a)  $5 \times 10^{15}$  boron atoms/cm<sup>3</sup>; (b)  $2 \times 10^{16}$  boron atoms/cm<sup>3</sup> and  $1.5 \times 10^{16}$  arsenic atoms/cm<sup>3</sup>; and (c)  $5 \times 10^{15}$  boron atoms/cm<sup>3</sup>,  $10^{17}$  arsenic atoms/cm<sup>3</sup>, and  $10^{17}$  gallium atoms/cm<sup>3</sup>.
- \* 5. Consider a compensated  $n$ -type silicon at  $T = 300$  K, with a conductivity of  $\sigma = 16$  ( $\Omega$ -cm)<sup>-1</sup> and an acceptor doping concentration of  $10^{17}$  cm<sup>-3</sup>. Determine the donor

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7. A four-point probe (with probe spacing of 0.5 mm) is used to measure the resistivity of a *p*-type silicon sample. Find the resistivity of the sample if its diameter is 200 mm and its thickness is 50  $\mu\text{m}$ . The contact current is 1 mA, and the measured voltage between the inner two probes is 10 mV.
8. Given a silicon sample of unknown doping, Hall measurement provides the following information:  $W = 0.05 \text{ cm}$ ,  $A = 1.6 \times 10^{-3} \text{ cm}^2$  (refer to Fig. 8),  $I = 2.5 \text{ mA}$ , and the magnetic field is 30 nT ( $1 \text{ T} = 10^{-4} \text{ Wb/cm}^2$ ). If a Hall voltage of +10 mV is measured, find the Hall coefficient, conductivity type, majority carrier concentration, resistivity, and mobility of the semiconductor sample.
9. A semiconductor is doped with  $N_D$  ( $N_D \gg n_i$ ) and has a resistance  $R_1$ . The same semiconductor is then doped with an unknown amount of acceptors  $N_A$  ( $N_A \gg N_D$ ), yielding a resistance of  $0.5 R_1$ . Find  $N_A$  in terms of  $N_D$  if  $D_n/D_p = 50$ .
- \*10. Consider a semiconductor that is nonuniformly doped with donor impurity atoms  $N_D(x)$ . Show that the induced electric field in the semiconductor in thermal equilibrium is given by  $\mathcal{E}(x) = -\left(\frac{kT}{q}\right) \frac{1}{N_D(x)} \frac{dN_D(x)}{dx}$ .

### FOR SECTION 3.2 CARRIER DIFFUSION

11. An intrinsic Si sample is doped with donors from one side such that  $N_D = N_0 \exp(-ax)$ .  
(a) Find an expression for the built-in field  $\mathcal{E}(x)$  at equilibrium over the range for which  $N_D \gg n_i$ . (b) Evaluate  $\mathcal{E}(x)$  when  $a = 1 \mu\text{m}^{-1}$ .
12. An *n*-type Si slice of a thickness  $L$  is inhomogeneously doped with phosphorus donor whose concentration profile is given by  $N_D(x) = N_0 + (N_L - N_0)(x/L)$ . What is the formula for the electric potential difference between the front and the back surfaces when the sample is at thermal and electric equilibria regardless of how the mobility and diffusivity varies with position? What is the formula for the equilibrium electric field at a plane  $x$  from the front surface for a constant diffusivity and mobility?

### FOR SECTION 3.3 GENERATION AND RECOMBINATION PROCESS

13. Calculate the electron and hole concentration under steady-state illumination in an *n*-type silicon with  $G_L = 10^{16} \text{ cm}^{-3}\text{s}^{-1}$ ,  $N_D = 10^{15} \text{ cm}^{-3}$ , and  $\tau_n = \tau_p = 10 \mu\text{s}$ .
14. An *n*-type silicon sample has  $2 \times 10^{16}$  arsenic atoms/cm<sup>3</sup>,  $2 \times 10^{15}$  bulk recombination centers/cm<sup>3</sup>, and  $10^{10}$  surface recombination centers/cm<sup>2</sup>. (a) Find the bulk minority carrier lifetime, the diffusion length, and the surface recombination velocity under low-injection conditions. The values of  $\sigma_p$  and  $\sigma_s$  are  $5 \times 10^{-15}$  and  $2 \times 10^{-16} \text{ cm}^2$ , respectively. (b) If the sample is illuminated with uniformly absorbed light that creates  $10^{17}$  electron-hole pairs/cm<sup>2</sup>-s, what is the hole concentration at the surface?
- \*15. Assume that an *n*-type semiconductor is uniformly illuminated, producing a uniform excess generation rate  $G$ . Show that in steady state the change in the semiconductor conductivity is given by  $\Delta\sigma = q(\mu_n + \mu_p)\tau_p G$ .

### FOR SECTION 3.4 CONTINUITY EQUATION

- 16. The total current in a semiconductor is constant and is composed of electron drift current and hole diffusion current. The electron concentration is constant and equal to  $10^{16} \text{ cm}^{-3}$ .

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The hole concentration is given by

$$p(x) = 10^{15} \exp\left(\frac{-x}{L}\right) \text{ cm}^{-3} \quad (x \geq 0),$$

where  $L = 12 \mu\text{m}$ . The hole diffusion coefficient is  $D_p = 12 \text{ cm}^2/\text{s}$  and the electron mobility is  $\mu_n = 1000 \text{ cm}^2/\text{V}\cdot\text{s}$ . The total current density is  $J = 4.8 \text{ A/cm}^2$ . Calculate (a) the hole diffusion current density versus  $x$ , (b) the electron current density versus  $x$ , and (c) the electric field versus  $x$ .

- \*17. Excess carriers are injected on one surface of a thin slice of  $n$ -type silicon with thickness  $W$  and extracted at the opposite surface where  $p_n(W) = p_{no}$ . There is no electric field in the region  $0 < x < W$ . Derive the expression for current densities at the two surfaces.
- 18. In Prob. 17, if carrier lifetime is  $50 \mu\text{s}$  and  $W = 0.1 \text{ mm}$ , calculate the portion of injected current that reaches the opposite surface by diffusion ( $D = 50 \text{ cm}^2/\text{s}$ ).
- \*19. An  $n$ -type semiconductor has excess carrier holes  $10^{14} \text{ cm}^{-3}$ , and a bulk minority carrier lifetime  $10^{-6} \text{ s}$  in the bulk material, and a minority carrier lifetime  $10^{-7} \text{ s}$  at the surface. Assume zero applied electric field and let  $D_p = 10 \text{ cm}^2/\text{s}$ . Determine the steady-state excess carrier concentration as a function of distance from the surface ( $x = 0$ ) of the semiconductor.

## FOR SECTION 3.5 THERMIONIC EMISSION PROCESS

- 20. A metal, with a work function  $\phi_m = 4.2 \text{ V}$ , is deposited on an  $n$ -type silicon semiconductor with affinity  $\chi = 4.0 \text{ V}$  and  $E_g = 1.12 \text{ eV}$ . What is the potential barrier height seen by electrons in the metal moving into the semiconductor?
- 21. Consider a tungsten filament with metal work function  $\phi_m$  inside a high vacuum chamber. Show that if a current is passed through the filament to heat it up sufficiently, the electrons with enough thermal energy will escape into the vacuum and the resulted thermionic current density is

$$J = A^* T^2 \exp\left(\frac{-q\phi_m}{kT}\right)$$

where  $A^*$  is  $4\pi qmk^2/h^3$  and  $m$  is free electron mass. The definite integral

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \left(\frac{\pi}{a}\right)^{1/2}.$$

## FOR SECTION 3.6 TUNNELING PROCESS

- \* 22. Consider a electron with an energy of  $2 \text{ eV}$  impinging on a potential barrier with  $20 \text{ eV}$  and a width of  $3 \text{ \AA}$ . What is the tunneling probability?
- 23. Evaluate the transmission coefficient for an electron of energy  $2.2 \text{ eV}$  impinging on a potential barrier of height  $6.0 \text{ eV}$  and thickness  $10^{-10} \text{ meters}$ . Repeat the calculation for a barrier thickness of  $10^{-9} \text{ meters}$ .

## FOR SECTION 3.7 HIGH FIELD EFFECTS

- \* 24. Use the velocity-field relations for Si and GaAs shown in Fig. 22 to determine the transit time of electrons through a  $1 \mu\text{m}$  distance in these materials for an electric field of (a)  $1 \text{ kV/cm}$  and (b)  $50 \text{ kV/cm}$ .
- \* 25. Assume that a conduction electron in Si ( $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$ ) has a thermal energy  $kT$ , related to its mean thermal velocity by  $E_{th} = m_0 v_{th}^2/2$ . This electron is placed in an electric field of  $100 \text{ V/cm}$ . Show that the drift velocity of the electron in this case is small compared to its thermal velocity. Repeat for a field of  $10^4 \text{ V/cm}$ , using the same value of  $\mu_n$ . Comment on the actual mobility effects at this higher value of field.

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## PROBLEMS (\* DENOTES DIFFICULT PROBLEMS)

### FOR SECTION 4.3 DEPLETION REGION

- \*1. A diffused silicon  $p$ - $n$  junction has a linearly graded junction on the  $p$ -side with  $a = 10^{19} \text{ cm}^{-4}$ , and a uniform doping of  $3 \times 10^{14} \text{ cm}^{-3}$  on the  $n$ -side. If the depletion layer width of the  $p$ -side is  $0.8 \mu\text{m}$  at zero bias, find the total depletion layer width, built-in potential, and maximum field at zero bias.
- \*2. Sketch the potential distribution in the Si  $p$ - $n$  junction in Prob. 1.
3. For an ideal silicon  $p$ - $n$  abrupt junction with  $N_A = 10^{17} \text{ cm}^{-3}$  and  $N_D = 10^{15} \text{ cm}^{-3}$ , (a) calculate  $V_{bi}$  at 250, 300, 350, 400, 450, and 500 K and plot  $V_{bi}$  versus  $T$ ; (b) comment on your result in terms of energy band diagram; and (c) find the depletion layer width and the maximum field at zero bias for  $T = 300 \text{ K}$ .
4. Determine the  $n$ -type doping concentration to meet the following specifications for a Si  $p$ - $n$  junction:  
 $N_A = 10^{18} \text{ cm}^{-3}$ ,  $\mathcal{E}_{\text{max}} = 4 \times 10^5 \text{ V/cm}$  at  $V_R = 30 \text{ V}$ ,  $T = 300 \text{ K}$ .

### FOR SECTION 4.4 DEPLETION CAPACITANCE

- \*5. An abrupt  $p$ - $n$  junction has a doping concentration of  $10^{15}$ ,  $10^{16}$ , or  $10^{17} \text{ cm}^{-3}$  on the lightly doped  $n$ -side and of  $10^{19} \text{ cm}^{-3}$  on the heavily doped  $p$ -side. Obtain series of curves of  $1/C^2$  versus  $V$ , where  $V$  ranges from  $-4 \text{ V}$  to  $0 \text{ V}$  in steps of  $0.5 \text{ V}$ . Comment on the slopes and the interceptions at the voltage axis of these curves.
6. For a silicon linearly graded junction with a impurity gradient of  $10^{20} \text{ cm}^{-4}$ , calculate the built-in potential and the junction capacitance at reverse bias of  $4 \text{ V}$  ( $T = 300 \text{ K}$ ).
- \*7. A one-sided  $p^+-n$  Si junction at  $300 \text{ K}$  is doped with  $N_A = 10^{19} \text{ cm}^{-3}$ . Design the junction so that  $C_j = 0.85 \text{ pF}$  at  $V_R = 4.0 \text{ V}$ .

### FOR SECTION 4.5 CURRENT-VOLTAGE CHARACTERISTICS

8. Assume that the  $p$ - $n$  junction considered in Prob. 3 contains  $10^{15} \text{ cm}^{-3}$  generation-recombination centers located  $0.02 \text{ eV}$  above the intrinsic Fermi level of silicon with  $\sigma_n = \sigma_p = 10^{-15} \text{ cm}^2$ . If  $v_{th} \cong 10^7 \text{ cm/s}$ , calculate the generation and recombination current at  $-0.5 \text{ V}$ .
9. Consider a Si  $p$ - $n$  junction with  $n$ -type doping concentration of  $10^{16} \text{ cm}^{-3}$  and is forward biased with  $V = 0.8 \text{ V}$  at  $300 \text{ K}$ . Calculate the minority-carrier hole concentration at the edge of the space charge region.

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10. Calculate the applied reverse-bias voltage at which the ideal reverse current in a *p-n* junction diode at  $T = 300$  K reaches 95% of its reverse saturation current value.
11. Design the Si *p-n* diode such that  $J_n = 25$  A/cm<sup>2</sup> and  $J_p = 7$  A/cm<sup>2</sup> at  $V_a = 0.7$  V. The remaining parameters are given in Ex. 5.
12. An ideal silicon *p-n* junction has  $N_D = 10^{18}$  cm<sup>-3</sup>,  $N_A = 10^{16}$  cm<sup>-3</sup>,  $\tau_p = \tau_n = 10^{-6}$  s, and a device area of  $1.2 \times 10^{-5}$  cm<sup>2</sup>. (a) Calculate the theoretical saturation current at 300 K. (b) Calculate the forward and reverse currents at  $\pm 0.7$  V.
13. In Prob. 12, assume the widths of the two sides of the junction are much greater than the respective minority-carrier diffusion length. Calculate the applied voltage at a forward current of 1 mA at 300 K.
14. A silicon *p-n* junction has the following parameters at 300 K:  $\tau_p = \tau_n = 10^{-6}$  s,  $N_D = 10^{15}$  cm<sup>-3</sup>,  $N_A = 10^{19}$  cm<sup>-3</sup>. (a) Plot diffusion current density,  $J_{gen}$ , and total current density versus applied reverse voltage. (b) Repeat the above results for  $N_D = 10^{17}$  cm<sup>-3</sup>.

#### FOR SECTION 4.6 CHARGE STORAGE AND TRANSIENT BEHAVIOR

15. For an ideal abrupt silicon *p-n* junction with  $N_D = 10^{16}$  cm<sup>-3</sup>, find the stored minority carriers per unit area in the neutral *n*-region when a forward bias of 1 V is applied. The length of neutral region is 1  $\mu$ m and the diffusion length of the holes is 5  $\mu$ m.

#### FOR SECTION 4.7 JUNCTION BREAKDOWN

16. For a silicon *p-n* one-sided abrupt junction with  $N_D = 10^{15}$  cm<sup>-3</sup>, find the depletion layer width at breakdown. If the *n*-region is reduced to 5  $\mu$ m, calculate the breakdown voltage and compare your result with Fig. 29.
17. Design an abrupt Si *p-n* junction diode that has a reverse breakdown voltage of 130 V and has a forward-bias current of 2.2 mA at  $V_a = 0.7$  volt. Assume  $\tau_{p0} = 10^{-7}$  s.
18. In Fig. 20b, the avalanche breakdown voltage increases with increasing temperature. Give a qualitative argument for the result.
19. If  $\alpha_n = \alpha_p = 10^4 (\mathcal{E}/4 \times 10^5)^6$  cm<sup>-1</sup> in gallium arsenide, where  $\mathcal{E}$  is in V/cm, find the breakdown voltage of (a) a *p-i-n* diode with an intrinsic-layer width of 10  $\mu$ m and (b) *p-n* junction with a doping of  $2 \times 10^{16}$  cm<sup>-3</sup> for the lightly doped side.
20. Consider that a Si *p-n* junction at 300 K with a linearly doping profile varies from  $N_A = 10^{18}$  cm<sup>-3</sup> to  $N_D = 10^{18}$  cm<sup>-3</sup> over a distance of 2  $\mu$ m. Calculate the breakdown voltage.

#### FOR SECTION 4.8 HETEROJUNCTION

21. For the ideal heterojunction in Ex. 10, find the electrostatic potential and depletion width in each material for applied voltage of 0.5 V and -5 V.
22. For an *n*-type GaAs/*p*-type Al<sub>0.3</sub>Ga<sub>0.7</sub>As heterojunction at room temperature,  $\Delta E_C = 0.21$  eV. Find the total depletion width at thermal equilibrium when both sides have impurity concentration of  $5 \times 10^{15}$  cm<sup>-3</sup>. (Hint: the bandgap of Al<sub>x</sub>Ga<sub>1-x</sub>As is given by  $E_g(x) = 1.424 + 1.247x$  eV, and the dielectric constant is  $12.4 - 3.12x$ . Assume  $N_C$  and  $N_V$  are the same for Al<sub>x</sub>Ga<sub>1-x</sub>As with  $0 < x < 0.4$ .)

ing above 10,000 V. We have considered the basic characteristic of thyristor operation. In addition, we discussed the bidirectional thyristor (diac and triac) that has on-off states with either positive or negative terminal voltages, the asymmetric thyristor that has very short turn-on and turn-off times, the gate turn-off thyristor (GTO), and the light-activated thyristor. Thyristors can cover a wide range of applications from low-frequency high-current power supplies to high-frequency, low-power applications, including lighting controls, home appliances, and industrial equipment.

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## PROBLEMS (\* INDICATES DIFFICULT PROBLEMS)

### FOR SECTION 5.2 STATIC CHARACTERISTICS OF BIPOLAR TRANSISTOR

1. An  $n$ - $p$ - $n$  transistor has a base transport factor  $\alpha_T$  of 0.998, an emitter efficiency of 0.997, and an  $I_{C0}$  of 10 nA. (a) Calculate  $\alpha_0$  and  $\beta_0$  for the device. (b) If  $I_B = 0$ , what is the emitter current?
2. Given that an ideal transistor has an emitter efficiency of 0.999 and the collector-base leakage current is 10  $\mu$ A, calculate the active region emitter current due to holes if  $I_B = 0$ .

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3. A silicon  $p-n-p$  transistor has impurity concentrations of  $5 \times 10^{18}$ ,  $2 \times 10^{17}$ , and  $10^{16} \text{ cm}^{-3}$  in the emitter, base, and collector, respectively. The base width is  $1.0 \mu\text{m}$ , and the device cross-sectional area is  $0.2 \text{ mm}^2$ . When the emitter-base junction is forward biased to  $0.5 \text{ V}$  and the base-collector junction is reverse biased to  $5 \text{ V}$ , calculate (a) the neutral base width and (b) the minority carrier concentration at the emitter-base junction.
4. For the transistor in Prob. 3, the diffusion constants of minority carriers in the emitter, base, and collector are  $52$ ,  $40$ , and  $115 \text{ cm}^2/\text{s}$ , respectively; and the corresponding lifetimes are  $10^{-8}$ ,  $10^{-7}$ , and  $10^{-6} \text{ s}$ . Find the current components  $I_{Ep}$ ,  $I_{Cp}$ ,  $I_{En}$ ,  $I_{Cn}$ , and  $I_{BB}$  illustrated in Fig. 5.
5. Using the results obtained from Prob. 3 and 4, (a) find the terminal currents  $I_E$ ,  $I_C$ , and  $I_B$  of the transistor; (b) calculate emitter efficiency, base transport factor, common-base current gain, and common-emitter current gain; and (c) comment on how the emitter efficiency and base transport factor can be improved.
6. Referring to the minority carrier concentration shown in Eq. 14, sketch  $p_n(x)/p_n(0)$  curves as a function of  $x$  with different  $W/L_p$ . Show that the distribution will approach a straight line when  $W/L_p$  is small enough (say  $W/L_p < 0.1$ ).
- \*7. For a transistor under the active mode of operation, use Eq. 14 to find the exact solutions of  $I_{Ep}$  and  $I_{Cp}$ .
8. Derive the expression for total excess minority-carrier charge  $Q_B$ , if the transistor is operated under the active mode and  $p_n(0) \gg p_{no}$ . Explain how the charge can be approximated by the triangle area in the base shown in Fig. 6. In addition, using the parameters in Prob. 3, find  $Q_B$ .
9. Using  $Q_B$  derived from Prob. 8, show that the collector current expressed in Eq. 27 can be approximated by  $I_C \approx (2D_p/W^2)Q_B$ .
10. Show that the base transport factor  $\alpha_T$  can be simplified to  $1 - (W^2/2L_p^2)$ .
- (B) 11. If the emitter efficiency is very close to unity, show that the common-emitter current gain  $\beta_0$  can be given by  $2L_p^2/W^2$ . (Hint: Use  $\alpha_T$  in Prob. 10.)
12. For a  $p^+-n-p$  transistor with high emitter efficiency, find the common-emitter current gain  $\beta_0$ . If the base width is  $2 \mu\text{m}$  and the diffusion constant of minority carrier in the base region is  $100 \text{ cm}^2/\text{s}$ , assume that the lifetime of the carrier in the base region is  $3 \times 10^{-7} \text{ s}$ . (Hint: Refer to  $\beta_0$  derived in Prob. 11.)
13. A silicon  $n-p-n$  bipolar transistor has impurity concentrations of  $3 \times 10^{18}$ ,  $2 \times 10^{16}$ , and  $5 \times 10^{15} \text{ cm}^{-3}$  in the emitter, base, and collector, respectively. Determine the diffusion constants of minority carrier in the three regions by using Einstein's relationship,  $D = (kT/q)\mu$ . Assume that the mobilities of electrons and holes,  $\mu_n$  and  $\mu_p$ , can be expressed as
 
$$\mu_n = 88 + \frac{1252}{(1 + 0.698 \times 10^{-17} N)} \quad \text{and} \quad \mu_p = 54.3 + \frac{407}{(1 + 0.374 \times 10^{-17} N)} \quad \text{at } T = 300 \text{ K}.$$
- \*14. Using the results obtained from Prob. 13, determine the current components in each region with  $V_{BE} = 0.6 \text{ V}$  (operated under active mode). The device cross-sectional area is  $0.01 \text{ mm}^2$  and the neutral-base width is  $0.5 \mu\text{m}$ . Assume the minority-carrier lifetime in each region is the same and equals to  $10^{-6} \text{ s}$ .
15. Based on the results obtain from Prob. 14, find the emitter efficiency, base transport factor, common-base current gain, and common-emitter current gain.
16. For an ion implanted  $n-p-n$  transistor the net impurity doping in the neutral base is given by  $N(x) = N_{AO}e^{-x/l}$ , where  $N_{AO} = 2 \times 10^{18} \text{ cm}^{-3}$  and  $l = 0.3 \mu\text{m}$ . (a) Find the total number of impurities in the neutral-base region per unit area (b) Find the average impurity concentration in the neutral-base region for a neutral-base width of  $0.8 \mu\text{m}$ .



17. Referring to Problem 16, if  $L_E = 1\mu\text{m}$ ,  $N_E = 10^{19}\text{cm}^{-3}$ ,  $D_E = 1\text{cm}^2/\text{s}$ , the average lifetime is  $10^{-6}\text{s}$  in the base, and the average diffusion coefficient in the base corresponds to the impurity concentration in Prob. 16, find the common-emitter current gain.
18. Estimate the collector current level for the transistor in Probs. 16 and 17 that has an emitter area of  $10^{-4}\text{cm}^2$ . The base resistance of the transistor can be expressed as  $10^{-3}\bar{\rho}_B/W$ , where  $W$  is the neutral-base width and  $\bar{\rho}_B$  is the average base resistivity.
- \*19. Plot the common-emitter current gain as a function of the base current  $I_B$  from 0 to  $25\mu\text{A}$  at a fixed  $V_{EC}$  of 5 V for the transistor shown in Fig. 10b. Explain why the current gain is not a constant.
20. The general equations of the emitter and collector currents for the basic Ebers-Moll model [J. J. Ebers and J. L. Moll, "Large-Signal Behavior of Junction Transistors," *Proc. IRE.*, 42, 1761(1954)] are

$$I_E = I_{FO}\left(e^{qV_{EB}/kT} - 1\right) - \alpha_R I_{RO}\left(e^{qV_{CB}/kT} - 1\right),$$

$$I_C = \alpha_F I_{FO}\left(e^{qV_{EB}/kT} - 1\right) - I_{RO}\left(e^{qV_{CB}/kT} - 1\right),$$

where  $\alpha_F$  and  $\alpha_R$  are the *forward common-base current gain* and the *reverse common-base current gain*, respectively.  $I_{FO}$  and  $I_{RO}$  are the saturation currents of the normally forward- and reverse-biased diodes, respectively. Find  $\alpha_F$  and  $\alpha_R$  in terms of the constants in Eqs. 25, 26, 28, and 29.

- \*21. Referring to the transistor in Example 2, find  $\alpha_F$ ,  $\alpha_R$ ,  $I_{FO}$ , and  $I_{RO}$  by using the equations derived in Problem 20.
22. Derive Eq. 32b for the collector current starting with the field-free steady-state continuity equation. (Hint: Consider the minority carrier distribution in the collector region.)

### FOR SECTION 5.3 FREQUENCY RESPONSE AND SWITCHING OF BIPOLAR TRANSISTOR

23. A Si transistor has  $D_p$  of  $10\text{cm}^2/\text{s}$  and  $W$  of  $0.5\mu\text{m}$ . Find the cutoff frequencies for the transistor with a common-base current gain  $\alpha_0$  of 0.998. Neglect the emitter and collector delays.
24. If we want to design a bipolar transistor with 5 GHz cutoff frequency  $f_T$ , what the neutral base width  $W$  will be? Assume  $D_p$  is  $10\text{cm}^2/\text{s}$  and neglect the emitter and collector delays.

### FOR SECTION 5.4 THE HETEROJUNCTION BIPOLAR TRANSISTOR

- \*25. Consider a  $\text{Si}_{1-x}\text{Ge}_x/\text{Si}$  HBT with  $x = 10\%$  in the base region (and 0% in emitter and collector region). The bandgap of the base region is 9.8% smaller than that of Si. If the base current is due to emitter injection efficiency only, what is the expected change in the common-emitter current gain between  $0^\circ$  and  $100^\circ\text{C}$ ?
- \*26. For an  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$  HBT, the bandgap of the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  is a function of  $x$  and can be expressed as  $1.424 + 1.247x\text{eV}$  (when  $x \leq 0.45$ ) and  $1.9 + 0.125x + 0.143x^2\text{eV}$  (when  $0.45 < x \leq 1$ ). Plot  $\beta_0(\text{HBT})/\beta_0(\text{BJT})$  as a function of  $x$ .

### FOR SECTION 5.5 THE THYRISTOR AND RELATED POWER DEVICES

27. For the doping profile shown in Fig. 22, find the width  $W$  ( $> 10\mu\text{m}$ ) of the  $n1$ -region so that the thyristor has a reverse blocking voltage of 120 V. If the current gain  $\alpha_2$  for the  $n1$ - $p2$ - $n2$  transistor is 0.4 independent of current, and  $\alpha_1$  of the  $p1$ - $n1$ - $p2$  transistor can be expressed as  $0.5\sqrt{L_p/W} \ln(j/j_0)$ , where  $L_p$  is  $25\mu\text{m}$  and  $j_0$  is  $5 \times 10^{-6}\text{A}/\text{cm}^2$ , find the cross-sectional area of the thyristor that will switch at a current  $I_s$  of 1 mA.
28. For a GTO thyristor shown in Fig. 32, find the minimum gate current  $I_g$  to turn off the thyristor. Assume the current gains of the  $p1$ - $n1$ - $p2$  and  $n1$ - $p2$ - $n2$  transistors are  $\alpha_1$  and  $\alpha_2$ , respectively.



10. Assume that the oxide trapped charge  $Q_{ot}$  has a density of  $5 \times 10^{11} \text{ cm}^{-2}$  locating solely at  $y = 5 \text{ nm}$ . The thickness of the oxide layer is  $10 \text{ nm}$ . Find the change in the flat-band voltage due to  $Q_{ot}$ .
11. Assume that the oxide trapped charge  $Q_{ot}$  in an oxide layer has a triangular distribution:  $\rho_{ot}(y) = q \times (5 \times 10^{23} \times y) \text{ cm}^{-3}$ . The thickness of oxide layer is  $10 \text{ nm}$ . Find the change in the flat-band voltage due to  $Q_{ot}$ .
12. Assume that initially there is a sheet of mobile ions at the metal-SiO<sub>2</sub> interface. After a long period of electrical stressing under a high positive gate voltage and raised temperature condition, the mobile ions completely drift to the SiO<sub>2</sub>-Si interface. This leads to a change of  $0.3 \text{ V}$  in the flat-band voltage. The thickness of oxide layer is  $10 \text{ nm}$ . Find the area density of  $Q_m$ .

#### FOR SECTION 6.2 MOSFET FUNDAMENTALS

13. Derive Eq. 34 from Eq. 33 in the text assuming  $V_D \ll (V_G - V_T)$ .
- \*14. Derive the  $I$ - $V$  characteristics of a MOSFET with the drain and gate connected together and the source and substrate grounded. Can one obtain the threshold voltage from these characteristics?
15. Consider a long-channel MOSFET with  $L = 1 \text{ }\mu\text{m}$ ,  $Z = 10 \text{ }\mu\text{m}$ ,  $N_A = 5 \times 10^{16} \text{ cm}^{-3}$ ,  $\mu_n = 800 \text{ cm}^2/\text{V}\cdot\text{s}$ ,  $C_o = 3.45 \times 10^{-7} \text{ F/cm}^2$ , and  $V_T = 0.7 \text{ V}$ . Find  $V_{Dsat}$  and  $I_{Dsat}$  for  $V_G = 5 \text{ V}$ .
16. Consider a submicron MOSFET with  $L = 0.25 \text{ }\mu\text{m}$ ,  $Z = 5 \text{ }\mu\text{m}$ ,  $N_A = 10^{17} \text{ cm}^{-3}$ ,  $\mu_n = 500 \text{ cm}^2/\text{V}\cdot\text{s}$ ,  $C_o = 3.45 \times 10^{-7} \text{ F/cm}^2$ , and  $V_T = 0.5 \text{ V}$ . Find the channel conductance for  $V_G = 1 \text{ V}$  and  $V_D = 0.1 \text{ V}$ .
17. For the device stated in Prob. 16, find the transconductance.
18. An  $n$ -channel,  $n^+$ -polysilicon-SiO<sub>2</sub>-Si MOSFET has  $N_A = 10^{17} \text{ cm}^{-3}$ ,  $Q_f/q = 5 \times 10^{10} \text{ cm}^{-2}$ , and  $d = 10 \text{ nm}$ . Calculate the threshold voltage.
19. For the device stated in Prob. 18, boron ions are implanted to increase the threshold voltage to  $+0.7 \text{ V}$ . Find the implant dose, assume that the implanted ions form a sheet of negative charges at the Si-SiO<sub>2</sub> interface.
20. A  $p$ -channel,  $n^+$ -polysilicon-SiO<sub>2</sub>-Si MOSFET has  $N_D = 10^{17} \text{ cm}^{-3}$ ,  $Q_f/q = 5 \times 10^{10} \text{ cm}^{-2}$ , and  $d = 10 \text{ nm}$ . Calculate the threshold voltage.
21. For the device stated in Prob. 20, boron ions are implanted to decrease the value of threshold voltage to  $-0.7 \text{ V}$ . Find the implant dose, assuming that the implanted ions form a sheet of negative charges at the Si-SiO<sub>2</sub> interface.
22. For the device stated in Prob. 20, if the  $n^+$  poly-Si gate is replaced by  $p^+$  poly-Si gate, what will the threshold voltage be?
23. A field transistor with a structure similar to Fig. 21 in the text has  $N_A = 10^{17} \text{ cm}^{-3}$ ,  $Q_f/q = 10^{11} \text{ cm}^{-2}$ , and an  $n^+$  polysilicon local interconnect as the gate electrode. If the requirement for sufficient isolation between device and well is  $V_T > 20 \text{ V}$ , calculate the minimum field oxide thickness.
24. A MOSFET has a threshold voltage of  $V_T = 0.5 \text{ V}$ , a subthreshold swing of  $100 \text{ mV/decade}$ , and a drain current of  $0.1 \text{ }\mu\text{A}$  at  $V_T$ . What is the subthreshold leakage current at  $V_G = 0$ ?
25. For the device stated in Prob. 24, calculate the reverse substrate-source voltage required to reduce the leakage current by one order of magnitude. ( $N_A = 5 \times 10^{17} \text{ cm}^{-3}$ ,  $d = 5 \text{ nm}$ ).

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## FOR SECTION 6.3 MOSFET SCALING

26. When the linear dimensions of MOSFET are scaled down by a factor of 10 based on the constant field scaling, what is the scaling factor for the corresponding switching energy?
- \*27. Based on the charge-sharing model, Fig. 24, show that the threshold voltage roll-off is given by Eq. 47.

## FOR SECTION 6.4 CMOS AND BiCMOS

28. Describe the pros and cons of BiCMOS.

## FOR SECTION 6.5 MOSFET ON INSULATOR

29. For an  $n$ -channel FD-SOI device having  $N_A = 5 \times 10^{17} \text{ cm}^{-3}$  and  $d = 4 \text{ nm}$ , calculate the maximum allowable thickness for Si channel layer ( $d_{Si}$ ).
30. For an  $n$ -channel SOI device with  $n^+$ -polysilicon gate having  $N_A = 5 \times 10^{17} \text{ cm}^{-3}$ ,  $d = 4 \text{ nm}$ , and  $d_{Si} = 30 \text{ nm}$ , calculate the threshold voltage. Assume that  $Q_f$ ,  $Q_{ot}$ , and  $Q_m$  are all zero.
31. For the device stated in Prob. 29, calculate the range of  $V_T$  distribution if the thickness variation of  $d_{Si}$  across the wafer is  $\pm 5 \text{ nm}$ .

## FOR SECTION 6.6 MOS MEMORY STRUCTURES

32. What is the capacitance of a DRAM capacitor if it is planar,  $1 \mu\text{m} \times 1 \mu\text{m}$ , with an oxide thickness of  $10 \text{ nm}$ ? Calculate the capacitance if the same surface area is used for a trench that is  $7 \mu\text{m}$  deep and with the same oxide thickness.
33. A DRAM must operate with a minimum refresh time of  $4 \text{ ms}$ . The storage capacitor in each cell has a capacitance of  $50 \text{ fF}$  and is fully charged to  $5 \text{ V}$ . Find the worst-case leakage current (i.e., during the refresh cycle 50% of the stored charge is lost) that the dynamic node can tolerate.
34. A floating-gate nonvolatile memory has an initial threshold voltage of  $-2 \text{ V}$ , and a linear-region drain conductance of  $10 \mu\text{mhos}$  at a gate voltage of  $-5 \text{ V}$ . After a write operating, the drain conductance increases to  $40 \mu\text{mhos}$  at the same gate voltage. Find the threshold voltage shift.

## FOR SECTION 6.7 THE POWER MOSFET

35. A power MOSFET has an  $n^+$ -polysilicon gate and a  $p$ -base with  $N_A = 10^{17} \text{ cm}^{-3}$ . Gate oxide thickness  $d = 100 \text{ nm}$ . Calculate the threshold voltage.
36. For the device stated in Prob. 35, calculate the effect of a positive fixed charge density of  $5 \times 10^{11} \text{ cm}^{-3}$  on the threshold voltage.

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## PROBLEMS (\* DENOTES DIFFICULT PROBLEMS)

### FOR SECTION 7.1 METAL-SEMICONDUCTOR CONTACTS

1. Calculate the theoretical barrier height and built-in potential in a metal-semiconductor diode for zero applied bias. Assume the metal work function is 4.55 eV, the electron affinity is 4.01 eV, and  $N_D = 2 \times 10^{16} \text{ cm}^{-3}$  at 300 K.
2. (a) Find the donor concentration and barrier height of the W-GaAs Schottky barrier diode shown in Fig. 6. (b) Compare the barrier height with that obtained from the saturation current density of  $5 \times 10^{-7} \text{ A/cm}^2$  shown in Fig. 8. (c) For a reverse bias of  $-1 \text{ V}$ , calculate the depletion-layer width  $W$ , the maximum field, and the capacitance.
3. Copper is deposited on a carefully prepared  $n$ -type silicon substrate to form an ideal Schottky diode.  $\phi_m = 4.65 \text{ eV}$ , the electron affinity is 4.01 eV,  $N_D = 3 \times 10^{16} / \text{cm}^3$ , and  $T = 300 \text{ K}$ . Calculate the barrier height, the built-in potential, the depletion-layer width, and the maximum field at a zero bias.
- \*4. The capacitance of a Au- $n$ -type GaAs Schottky barrier diode is given by the relation  $1/C^2 = 1.57 \times 10^5 - 2.12 \times 10^5 V_a$ , where  $C$  is expressed in  $\mu\text{F}$  and  $V_a$  is in volts. Taking the diode area to be  $10^{-1} \text{ cm}^2$ , calculate the built-in potential, the barrier height, the dopant concentration, and the work function.
5. Calculate the value of  $V_{bi}$  and  $\phi_m$  in an ideal metal-Si Schottky barrier contact. Assume the barrier height is 0.8 eV,  $N_D = 1.5 \times 10^{16} \text{ cm}^{-3}$ , and  $q\chi = 4.01 \text{ eV}$ .
6. In a metal-Si Schottky barrier contact, the barrier height is 0.75 eV and  $A^* = 110 \text{ A/cm}^2 \cdot \text{K}^2$ . Calculate the ratio of the injected hole current to the electron current at 300 K, assuming  $D_p = 12 \text{ cm}^2 \text{ s}^{-1}$ ,  $L_p = 1 \times 10^{-3} \text{ cm}$ , and  $N_D = 1.5 \times 10^{16} \text{ cm}^{-3}$ .

### FOR SECTION 7.2 MESFET

7. Given  $\phi_{Bn} = 0.9 \text{ eV}$  and  $N_D = 10^{17} \text{ cm}^{-3}$ , find the minimum value of the thickness of the epitaxial layer for a GaAs MESFET to be a depletion mode device (i.e.,  $V_T < 0$ ).
8. Assume the doping in a GaAs MESFET is  $N_D = 7 \times 10^{16} \text{ cm}^{-3}$  and the dimensions are  $a = 0.3 \mu\text{m}$ ,  $L = 1.5 \mu\text{m}$ ,  $Z = 5 \mu\text{m}$ ,  $\mu_n = 4500 \text{ cm}^2/\text{V}\cdot\text{s}$ , and  $\phi_{Bn} = 0.89 \text{ V}$ . Calculate the ideal value of  $g_m$  for  $V_G = 0$ , and  $V_D = 1 \text{ V}$ .
- \* 9. The  $n$ -channel GaAs MESFET shown in Fig. 10 has a barrier height  $\phi_{Bn} = 0.9 \text{ V}$ ,  $N_D = 10^{17} \text{ cm}^{-3}$ ,  $a = 0.2 \mu\text{m}$ ,  $L = 1 \mu\text{m}$ , and  $Z = 10 \mu\text{m}$ . (a) Is this an enhancement or depletion-

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- mode device? (b) Find the threshold voltage (the enhancement mode indicates  $V_T > 0$ ; the depletion mode indicates  $V_T < 0$ ).
10. An  $n$ -channel GaAs MESFET has a channel doping  $N_D = 2 \times 10^{15} \text{ cm}^{-3}$ ,  $\phi_{Bn} = 0.8 \text{ V}$ ,  $a = 0.5 \text{ } \mu\text{m}$ ,  $L = 1 \text{ } \mu\text{m}$ ,  $\mu_n = 4500 \text{ cm}^2/\text{V}\cdot\text{s}$ , and  $Z = 50 \text{ } \mu\text{m}$ . Find the pinch-off potential, threshold voltage, and the saturation current at  $V_G = 0$ .
11. The barrier height  $\phi_{Bn}$  of two GaAs  $n$ -channel MESFETs are the same and equal to  $0.85 \text{ V}$ . The channel doping in device 1 is  $N_D = 4.7 \times 10^{16} \text{ cm}^{-3}$ , and that in device 2 is  $N_D = 4.7 \times 10^{17} \text{ cm}^{-3}$ . Determine the channel thickness required in each device such that the threshold voltage is zero for each device.

## FOR SECTION 7.3 MODFET

12. For an abrupt AlGaAs/GaAs heterojunction with the  $n$ -AlGaAs layer doped to  $3 \times 10^{18} \text{ cm}^{-3}$ , the Schottky barrier is  $0.89 \text{ V}$  and the heterojunction conduction-band edge discontinuity  $\Delta E_C$  is  $0.23 \text{ eV}$ . Calculate the thickness of the doped AlGaAs layer  $d_1$  so that the threshold voltage is  $-0.5 \text{ V}$ . Assume the permittivity of the AlGaAs is  $12.3$ .
13. Find the thickness of the undoped spacer layer  $d_0$ , such that the two-dimensional electron gas concentration of an AlGaAs/GaAs heterojunction is  $1.25 \times 10^{12} \text{ cm}^{-2}$  at zero gate bias. Assume that the  $n$ -AlGaAs is doped to  $1 \times 10^{18} \text{ cm}^{-3}$  and has a thickness  $d_1$  of  $50 \text{ nm}$ , the Schottky barrier height is  $0.89 \text{ V}$ , and  $\Delta E_C/q = 0.23 \text{ V}$ . The permittivity of the AlGaAs is  $12.3$ .
14. Consider a AlGaAs/GaAs HFET with a  $50 \text{ nm}$   $n$ -AlGaAs and a  $10 \text{ nm}$  undoped AlGaAs spacer. Assume the threshold voltage is  $-1.3 \text{ V}$ ,  $N_D$  is  $5 \times 10^{17} \text{ cm}^{-3}$ ,  $\Delta E_C = 0.25 \text{ eV}$ , the channel width is  $8 \text{ nm}$ , and the permittivity of AlGaAs is  $12.3$ . Calculate the Schottky barrier height and the two-dimensional electron gas concentration at  $V_G = 0$ .
15. The AlGaAs/GaAs has a two-dimensional electron gas concentration of  $1 \times 10^{12} \text{ cm}^{-2}$ , the spacer is  $5 \text{ nm}$ , the channel width is  $8 \text{ nm}$ , the pinch-off voltage is  $1.5 \text{ V}$ ,  $\Delta E_C/q = 0.23 \text{ V}$ , and the doping concentration of AlGaAs is  $10^{18} \text{ cm}^{-3}$ . The Schottky barrier height is  $0.8 \text{ V}$ . Find the thickness of the doped AlGaAs and the threshold voltage.
16. Consider an  $n$ -AlGaAs-intrinsic GaAs abrupt heterojunction. Assume that the AlGaAs is doped to  $N_D = 3 \times 10^{18} \text{ cm}^{-3}$  and has a thickness of  $35 \text{ nm}$  (there is no spacer). Let  $\phi_{Bn} = 0.89 \text{ V}$ , and assume that  $\Delta E_C = 0.24 \text{ eV}$  and the dielectric constant is  $12.3$ . Calculate (a)  $V_p$  and (b)  $n_s$  for  $V_G = 0$ .

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## PROBLEMS (\* DENOTES DIFFICULT PROBLEMS)

### FOR SECTION 8.1 BASIC MICROWAVE TECHNOLOGY

1. For a nearly lossless transmission line ( $R$  is very small) with a  $75 \Omega$  characteristic impedance, if this transmission line has a unit length capacitance of  $2 \text{ pF}$ , what is the unit length inductance of this transmission line?
2. For a cavity of the dimensions  $a = 10 \text{ cm}$  ( $0.1 \text{ m}$ ),  $b = 5 \text{ cm}$  ( $0.05 \text{ m}$ ), and  $d = 25 \text{ cm}$  ( $0.25 \text{ m}$ ), find the resonant frequency in the dominant  $TE_{101}$  mode.

### FOR SECTION 8.2 TUNNEL DIODE

3. Find the depletion-layer capacitance and depletion-layer width at  $0.25 \text{ V}$  forward bias for a GaAs tunnel diode doped to  $10^{19} \text{ cm}^{-3}$  on both sides, using the abrupt junction approximation and assuming  $V_n = V_p = 0.03 \text{ V}$ .
4. The current-voltage characteristic of a GaSb tunnel diode can be expressed by the empirical form of Eq. 5, with  $I_p = 10 \text{ mA}$ ,  $V_p = 0.1 \text{ V}$ , and  $I_0 = 0.1 \text{ nA}$ . Find the largest negative differential resistance and the corresponding voltage.

### FOR SECTION 8.3 IMPATT DIODE

5. The variation of electric field in the depletion region due to avalanche-generated space charge gives rise to an incremental resistance for abrupt  $p^+-n$  diode. The incremental resistance is called the space-charge resistance,  $R_{SC}$ , and is given by  $(1/I) \int_0^W \Delta \mathcal{E} dx$ , where  $\Delta \mathcal{E}$  is given by

$$\Delta \mathcal{E}(W) = \frac{\int_0^W \rho_s dx}{\epsilon_s} = \frac{IW}{A\epsilon_s v_s}$$

- (a) Find  $R_{SC}$  for a  $p^+-n$  Si IMPATT diode with  $N_D = 10^{15} \text{ cm}^{-3}$ ,  $W = 12 \mu\text{m}$ , and  $A = 5 \times 10^{-4} \text{ cm}^2$ . (b) Find the total applied dc voltage for a current density of  $10^3 \text{ A/cm}^2$ .
6. A GaAs IMPATT diode is operated at  $10 \text{ GHz}$  with a dc bias of  $100 \text{ V}$  and an average biasing current ( $I_0/2$ ) of  $100 \text{ mA}$ . (a) If the power-generating efficiency is  $25\%$  and the thermal resistance of the diode is  $10^\circ \text{ C/W}$ , find the junction temperature rise above the

16

room temperature. (b) If the breakdown voltage increases with temperature at a rate of 60 mV/°C, find the breakdown voltage of the diode at room temperature.

7. Consider a GaAs single drift lo-hi-lo IMPATT diode shown in Fig. 6c with an avalanche region width (where the electric field is constant) of 0.4  $\mu\text{m}$  and a total depletion width of 3  $\mu\text{m}$ . The  $n^+$  clump has a charge  $Q$  of  $1.5 \times 10^{12}/\text{cm}^2$ . (a) Find the breakdown voltage of the diode and the maximum field at breakdown. (b) Is the field in the drift region high enough to maintain the velocity saturation of electrons? (c) Find the operating frequency.
- \*8. A silicon  $n^+p\text{-}\pi\text{-}p^+$  IMPATT diode has a  $p$ -layer 3  $\mu\text{m}$  thick and a  $\pi$ -layer (low-doping  $p$ -layer) 9  $\mu\text{m}$  thick. The biasing voltage must be high enough to cause avalanche breakdown in the  $p$ -region and velocity saturation in the  $\pi$  region. (a) Find the minimum required biasing voltage and the doping concentration of the  $p$ -region. (b) Estimate the transit time of the device.

#### FOR SECTION 8.4 TRANSFERRED-ELECTRON DEVICES

9. An InP TED is 1  $\mu\text{m}$  long with a cross-section area of  $10^{-4} \text{ cm}^2$  and is operated in the transit-time mode. (a) Find the minimum electron density  $n_0$  required for transit-time mode. (b) Find the time between current pulses. (c) Calculate the power dissipated in the device if it is biased at one-half the threshold.
- \*10. (a) Find the effective density of states in the upper valley  $N_{CU}$  of the GaAs conduction band. The upper-valley effective mass is  $1.2 m_0$ . (b) The ratio of electron concentrations between the upper and lower valleys is given by  $(N_{CU}/N_{CL}) \exp(-\Delta E/kT_e)$ , where  $N_{CL}$  is the effective density of states in the lower valley,  $\Delta E = 0.31 \text{ eV}$  is the energy difference, and  $T_e$  is the effective electron temperature. Find the ratio at  $T_e = 300 \text{ K}$ . (c) When electrons gain kinetic energies from the electric field,  $T_e$  increases. Find the concentration ratio for  $T_e = 1500 \text{ K}$ .

#### FOR SECTION 8.5 QUANTUM-EFFECT DEVICES

11. Molecular beam epitaxy interfaces are typically abrupt to within one or two monolayers (one monolayer = 0.28 nm in GaInAs) because of terrace formation in the growth plane. Estimate the energy level broadening for the ground and first excited-electron states of a 15 nm GaInAs quantum well bound by thick AlInAs barriers. (Hint: assume the case of two-monolayer thickness fluctuation and an infinity deep quantum well. The electron effective mass in GaInAs is  $0.0427 m_0$ .)
12. Find the first excited level and the corresponding width of the peak  $\Delta E_2$  for a RTD with AlAs (2 nm)/GaAs (6.78 nm)/AlAs (2 nm). If we want to maintain the same energy level but increase the width  $\Delta E_2$  by a factor of 10, what should be the thicknesses of AlAs and GaAs?