CHAPTER 14

Noise in Active Networks

14.1 INTRODUCTION

Having done a stability check and having met the gain requirements of an amplifier, the next important point to consider is noise. In an RF/microwave amplifier, the existence of noise signal plays an important role in the overall design procedure and needs to be grasped before a meaningful design process can be developed.

Noise power results from random processes that exist in nature. These random processes can be classified in several important classes each generating a certain type of noise which will be characterized shortly.

Some of the most important types of random processes are:

1. *Thermal vibrations of atoms, electrons and molecules in a component at any temperature above 0 °K.*
2. *Flow of charges (electrons and/or holes) in a wire or a device.*
3. *Emission of charges (electrons or ions) from a surface such as the cathode of a diode or an electron tube, etc.*
4. *Wave propagation through atmosphere or any other gas*

14.2 IMPORTANCE OF NOISE

Noise is passed into a microwave component or system either from an external source or is generated within the unit itself. Regardless of the manner of entrance of the noise signal, the noise level of a system greatly affects the performance of the system by setting the minimum detectable signal in the presence of noise. Therefore it is often desirable to reduce the influence of external noise signals and minimize the generation of noise signals within the unit, in order to achieve the best performance.

14.3 NOISE DEFINITION

Since noise considerations are of important consequences, we need to define it first:

DEFINITION- ELECTRICAL NOISE (OR NOISE): *Is defined to be any unwanted electrical disturbance or spurious signal. These unwanted signals are random in nature and are generated either internally in the electronic components or externally through impinging electromagnetic radiation.*

Since signals are totally random and uncorrelated in time, they are best analyzed though statistical methods. Their statistical properties can be briefly summarized as:

a. The "Mean value" of the noise signal is zero, i.e.,

$$\overline{\mathbf{V}_n} = \mathbf{Lim}_{T\to\infty} \mathbf{1/T} \int_{t_1}^{t_1+T} \mathbf{V_n(t)dt = 0} \qquad (14.1)$$

Where $\overline{V_n}$ is the noise mean value, $V_n(t)$ is the instantaneous noise voltage, t_1 is any arbitrary point in time and T is any arbitrary period of time ideally a large one approaching ∞.

b. The "mean-square-value" of the noise signal is a constant value, i.e.,

$$\overline{\mathbf{V}_n^{\,2}} = \mathbf{Lim}_{T\to\infty} \mathbf{1/T} \int_{t_1}^{t_1+T} [\mathbf{V_n(t)}]^2 \mathbf{dt = Constant} \qquad (14.2)$$

c. The "root-mean-square" (rms) of a noise signal is given by:

$$\mathbf{V}_{n,rms} = \sqrt{\overline{\mathbf{V}_n^{\,2}}} \qquad (14.3)$$

or,

$$(\mathbf{V}_n)^2_{rms} = \overline{\mathbf{V}_n^{\,2}} \qquad (14.4)$$

The concept of "root-mean-square value" of noise as given by Equation (14.3), is based on the fact that the "mean-square value", $\overline{\mathbf{V}_n^{\,2}}$, is proportional to the "noise power". Thus if we take the square root of Equation (14.2), we obtain the "rms value" of the noise voltage which is the "effective value" of the noise voltage.

14.4 SOURCES OF NOISE

There are several types of noise which needs to be defined:

a. DEFINITION-THERMAL NOISE (ALSO CALLED JOHNSON NOISE OR NYQUIST NOISE): *is the most basic type of noise which is caused by thermal vibration of bound charges and thermal agitation of electrons in a conductive material. This is common to all passive or active devices.*

b. DEFINITION-SHOT NOISE (OR SCHOTTKY NOISE): *is caused by random passage of discrete charge carriers (causing a current "I", due to motion of electrons or holes) in a solid state device while crossing a junction or other discontinuities. It is commonly found in a semiconductor device (e.g. in a pn junction of a diode or a transistor) and is proportional to* $(I)^{1/2}$.

c. DEFINITION-FLICKER NOISE (ALSO CALLED 1/f NOISE): *is small vibrations of a current due to the following factors:*

1. *Random injection or recombination of charge carriers at an interface, such as at a metal semiconductor interface (in semiconductor devices).*
2. *Random charges in cathode emissions of electric charges such as at a cathode-air interface (in a thermionic tube).*

Flicker noise is a frequency-dependent noise, which distorts the signal by adding more noise to the lower part of the signal band than the upper part. It exists at lower frequencies, almost from DC extending down to approximately 500 kHz to 1 MHz at a rate of –10 dB per decade.

14.5 THERMAL NOISE ANALYSIS

To analyze noise, let us consider the circuit shown in Figure 14.1a where a noisy resistor is connected to the input port of a two-port network. Focusing primarily on thermal noise, we note that the available noise power (i.e. maximum power available under matched conditions) from any arbitrary resistor has been shown by Nyquist to be:

$$P_N = kTB \qquad (14.5)$$

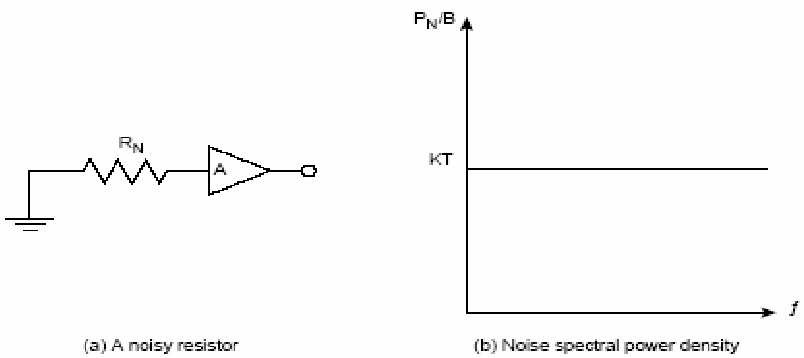
Where,

k= Boltzmann's constant (=1.374x10^{-23} J/K).
T= The resistor's physical temperature.
B= The 2-port network's bandwidth (i.e., B= f_H-f_L).

Since the noise power does not depend on the center frequency of operation but only on the bandwidth, it is called "white noise" as shown in Figure 14.1b.

There are a few observations about noise power (P_N) which is worth considering:

a. As bandwidth (B) is reduced, so does the noise power which means narrower bandwidth amplifiers are less noisy,



(a) A noisy resistor (b) Noise spectral power density

FIGURE 14.1 White noise in an amplifier.

b. As temperature (T) is reduced, the noise power is also lessened which means cooler devices or amplifiers generate less noise power,
c. Increasing bandwidth to infinity causes an infinite noise power (called ultraviolet catastrophe) which is incorrect since (14.5) for noise power is only valid up to approximately 1000 GHz.

14.6 NOISE MODEL OF A NOISY RESISTOR

A noisy resistor (R_N) at a temperature (T) can be modeled by an ideal noiseless resistor (R_{NO}) at 0 °K in conjunction with a noise voltage source ($V_{n,rms}$) as shown Figure 14.2. If we assume that the resistor value is independent of temperature then $\mathbf{R_{NO}=R_N}$.

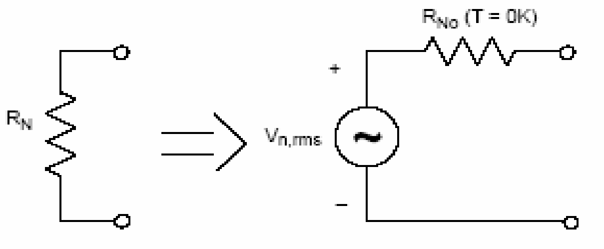
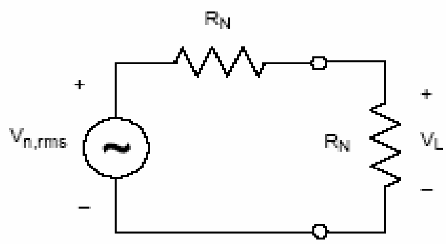


FIGURE 14.2 Model of a noisy resistor.

From this model, the available noise power to the load (under matched condition) is given by (see Figure 14.3):

$$\mathbf{P_N = \frac{V_{n,rms}^{\ 2}}{4R_N}} \qquad (14.6)$$

FIGURE 14.3 Available noise power.



Equation (14.17) provides the noise power available from a noisy resistor which equals Equation (14.5) for any arbitrary resistor. Thus:

$$\mathbf{P_N=kTB} \qquad (14.7a)$$

$$\mathbf{V_{n,rms} = 2\sqrt{P_N R_N} = 2\sqrt{kTBR_N}} \qquad (14.7b)$$

From Equation (14.7) we can observe that the noise voltage is proportional to ($R^{1/2}$). Thus higher-valued resistors have higher noise voltage even though they provide the same noise power level as the lower-valued resistors.

EXAMPLE 14.1

Calculate the noise power (in dBm) and rms noise voltage at T=290 °K for
a. R_N=1 Ω, B=1 Hz

b. R_N=2 MΩ, B=5 kHz.

Solution:

a. The noise power is given by:

k=1.374x10^{-23} J/°K

P_N=kTB=1.374x10^{-23} x290x1=3.985x10^{-21} W

Or in dBm, we have:

P_N(dBm)=10log(3.985x10^{-21}/10^{-3})=-174 dBm

This is the power per unit Hz. The corresponding noise voltage for a 1 Ω resistor is given by:

$$\mathbf{V_{n,rms} = 2\sqrt{P_N R_N} = 2\sqrt{3.985x10^{-21}x1} = 12.6x10^{-11} V}$$

=12.6x10^{-5} μV

b. For a 5 kHz bandwidth, we have

P_N=kTB=3.985x10^{-21}x5000=19.925x10^{-18} W

The corresponding noise voltage for a 2 MΩ resistor is given by

$$\mathbf{V_{n,rms} = 2\sqrt{P_N R_N} = 2\sqrt{19.925x10^{-18}x2x10^{6}} = 12.6x10^{-6} V}$$

=12.6 μV

14.7 EQUIVALENT NOISE TEMPERATURE

Any type of noise, in general, has a power spectrum which can be plotted in the frequency domain. If the noise power spectrum is not a strong function of frequency (i.e., it is "White" noise) then it can be modeled as an equivalent thermal noise source characterized by an "equivalent noise temperature" (T_e).

To define "the equivalent noise temperature" (Te), we consider an arbitrary white noise source with an available power (P_S) having a noiseless source resistance (R_S) as shown in Figure 14.4a. This white noise source can be replaced by a noisy resistor with an equivalent noise temperature (T_e) defined by:

$$\mathbf{T_e = \frac{P_S}{kB}} \qquad (14.8)$$

Where B is the bandwidth of the system or the component under consideration.

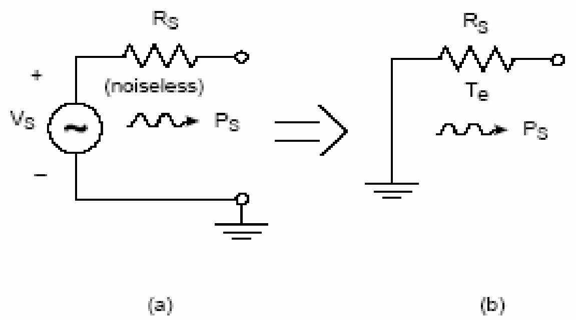


FIGURE 14.4 (a) An arbitrary white noise source, (b) equivalent circuit.

EXAMPLE 14.2:

Consider a noisy amplifier with available power gain (G_A) and bandwidth (B) connected to a source and load resistance (R) both at $T=T_S$ as shown in Figure 14.5. Determine the overall noise temperature of the combination and the total output noise power if the amplifier all by itself creates an output noise power of P_n.

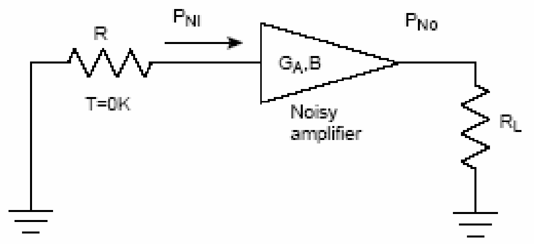


FIGURE 14.5 A noisy amplifier.

Solution:

To simplify the analysis, let us first assume that the source resistor is at T=0°K. This means that no noise enters the amplifier, i.e., $P_{Ni}=0$.

The noisy amplifier can be modeled by a noiseless amplifier with an input resistor at an equivalent noise temperature of:

$$\mathbf{T_e = \frac{P_n}{G_A kB}} \qquad (14.9)$$

T_e is called the equivalent noise temperature of the amplifier "referred to the input" as shown in Figure 14.6.

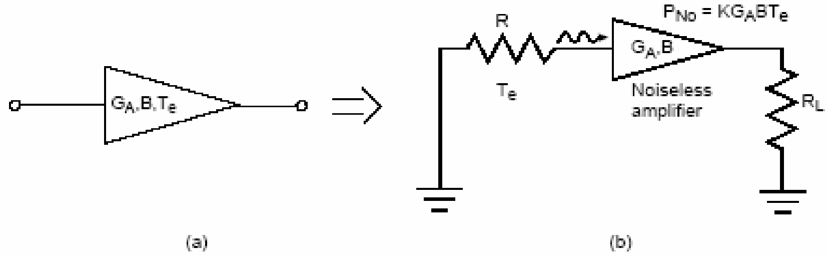


FIGURE 14.6 Equivalent models of a noisy amplifier.

Since source resistor (R) is at a physical temperature other than zero ($T=T_S$) , then as a result the combined equivalent noise temperature (T_e') is the addition of the two noise temperatures:

$$\mathbf{T_e' = T_e + T_S} \qquad (14.10)$$

Assuming the noise power at the input terminals of the amplifier is P_{NI} (=kT_SB), the total output noise power due to the amplified input thermal noise power will be (G_AP_{Ni}) which adds to the amplifier's generated noise power (P_n) linearly by using the superposition theorem (see Figure 14.7), i.e.,

$$\mathbf{P_{No,tot} = G_AP_{Ni} + P_n = G_AkB(T_S + T_e)}$$

$$\mathbf{P_{No,tot} = G_AkBT_e'} \qquad (14.11)$$

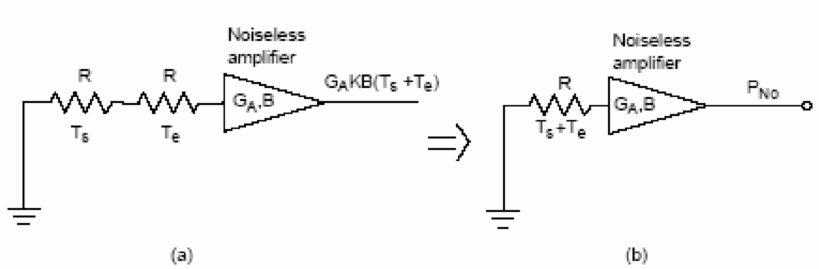


FIGURE 14.7 Total output noise power and its equivalent circuit.

NOTE: *It is important to note that from (14.11), the "equivalent noise temperature" (T_e') is defined by "referring" the total output noise power to the input port. Thus the same noise power is delivered to the load by driving a "noiseless amplifier" with a resistor at an equivalent temperature ($T_e' = T_e + T_S$).*

14.7.1 A Measurement Application: Y-Factor Method

The concept of equivalent noise temperature is commonly used in the measurement of noise temperature of an unknown amplifier using the "Y-factor method". In this method, the physical temperature of a matched resistor is changed to two distinct and known values:

a. One temperature (T_1) is at boiling water (T_1=100°C) or at room temperature (T_1=290 °K),
b. The second temperature (T_2) is obtained by using either a noise source (hotter source than room temperature) or a load immersed in liquid nitrogen at T=77 °K (a colder source than room temperature) as shown in Figure 14.8.

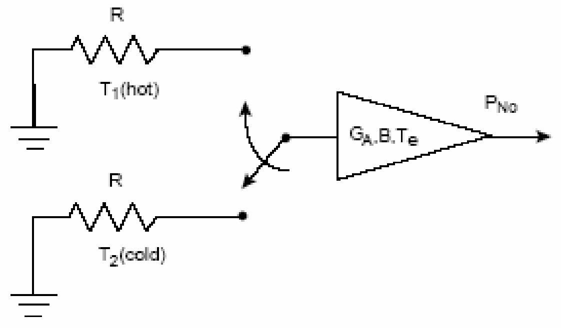


FIGURE 14.8 Y-factor method.

The amplifier's unknown noise temperature (T_e) can be obtained as follows:

$$\mathbf{P_{No,1} = G_A kB(T_1 + T_e)} \qquad (14.12)$$

$$\mathbf{P_{No,2} = G_A kB(T_2 + T_e)} \qquad (14.13)$$

Now define:

$$\mathbf{Y \equiv \frac{P_{No,1}}{P_{No,2}}}$$

Thus we can write:

$$\mathbf{Y=\frac{T_1+T_e}{T_2+T_e}} \tag{14.14}$$

Or,

$$\mathbf{T_e=\frac{T_1-YT_2}{Y-1}} \tag{14.15}$$

From a measurement of T_1, T_2 and Y the unknown amplifier's noise temperature (T_e) can be found.

POINT OF CAUTION: *To obtain an accurate value for Y, the two temperatures ideally must be far apart; otherwise Y≈1 and the denominator of Equation (14.15) will create relatively inaccurate results.*

NOTE: *A noise source "hotter" than room temperature, as used in the Y-factor measurement, would be a solid state noise source (such as an IMPATT diode) or a noise tube. Such active sources, providing a calibrated and specific noise power output in a particular frequency range, are most commonly characterized by their "excess noise ratio" values vs. frequency. The term "excess noise ratio" or ENR is defined as:*

$$\mathbf{ENR(dB)=10\log_{10}\left(\frac{P_N-P_O}{P_O}\right)=10\log_{10}\left(\frac{T_N-T_O}{T_O}\right)} \tag{14.16}$$

where P_N and T_N are the noise power and equivalent noise temperature of the active noise generator, and P_O and T_O are the noise power and temperature of a passive source(e.g. a matched load), respectively.

14.8 DEFINITIONS OF NOISE FIGURE

As discussed earlier, a noisy amplifier can be characterized by an equivalent noise temperature (T_e). An alternate method to characterize a noisy amplifier, is through the use of the concept of noise Figure which we need to define first.

DEFINITION- NOISE FIGURE: *Is defined to be the ratio of the total available noise power at the output, $(P_O)_{tot}$, to the output available noise power $(P_O)_i$ due to thermal noise coming only from the input resistor at the standard room temperature (T_O=290 °K).*

To formulate an equation for noise figure (F), let us transfer the noise generated inside the amplifier (P_n) to its input terminals and model it as a "noiseless" amplifier which is connected to a noisy resistor (R) at noise temperature (T_e) in series to another resistor (R) at $T=T_O$, both connected at the input terminals of the "noiseless" amplifier as shown in Figure 14.9. From this configuration we can write:

$$\mathbf{P_n=G_AkT_eB} \tag{14.17a}$$

$$\mathbf{(P_O)_i=G_AP_{Ni}=G_AkBT_O} \tag{14.17b}$$

$$\mathbf{(P_O)_{tot}=P_{NO}=Pn+(P_O)_i} \tag{14.18}$$

$$\mathbf{F=\frac{(P_O)_{tot}}{(P_O)_i}=\frac{(P_O)_i+P_n}{(P_O)_i}=1+\frac{P_n}{G_AP_{Ni}}} \tag{14.19a}$$

Or,

$$\mathbf{F = 1 + \frac{T_e}{T_O}} \qquad (14.19b)$$

Or, in dB we can write:

$$\mathbf{F = 10\log_{10}\left(1 + \frac{T_e}{T_O}\right)} \qquad (14.20)$$

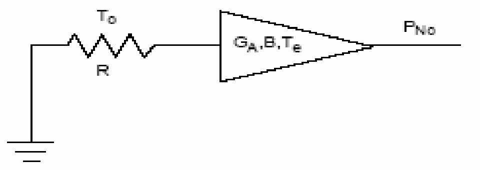


FIGURE 14.9 A noisy amplifier

From (14.19) we can see that "F" is bounded by:

$$\mathbf{1 \leq F \leq \infty} \qquad (14.21)$$

The lower boundary (F=1) is the best case scenario and is the noise Figure of an ideal noiseless amplifier where T_e=0.

From Equation (14.19b), we can write:

$$\mathbf{T_e = (F-1)T_O} \qquad (14.22)$$

NOTE1: *Temperature (T_e) is the equivalent noise temperature of the amplifier referred to the input.*

NOTE 2: *Either F or T_e can interchangeably be used to describe the noise properties of a two-port network. However, For small noise Figure values (i.e., when F≈1), use of T_e becomes preferable.*

POINT OF CAUTION: *It is interesting to note that the noise Figure is defined with reference to a matched input termination at room temperature (T_O=290 °K). Therefore if the physical temperature of the amplifier changes to some value other than T_O, we still use the room temperature (T_O=290 °K) to find the noise figure value.*

14.8.1 Alternate Definition of Noise Figure

From Equations (14.17) and (14.18), we can write:

$$\mathbf{P_{NO} = G_A P_{Ni} + P_n} \qquad (14.23)$$

$$\mathbf{(Po)_i = G_A P_{Ni}} \qquad (14.24)$$

Where $\mathbf{P_n = G_A k T_e B}$ is the generated noise power inside the amplifier. The noise figure can now be written as:

$$\mathbf{F = \frac{P_{N_O}}{(P_O)_i} = \frac{P_{N_O}}{G_A P_{N_i}} = 1 + \frac{P_n}{G_A P_{N_i}}} \qquad (14.25)$$

The available power gain (G_A) by definition is given by:

$$G_A = \frac{P_{S_o}}{P_{S_i}}$$

where P_{S_o} and P_{S_i} are the available signal power at the output and the input, respectively. Thus Equation (14.25) can now be written as:

$$F = \frac{P_{S_i}/P_{N_i}}{P_{S_o}/P_{N_o}} = \frac{(SNR)_i}{(SNR)_o} \qquad (14.26)$$

where $(SNR)_i$ and $(SNR)_o$ are the available signal-to-noise ratio at the input and output ports, respectively.

Equation (14.26) indicates that the noise figure can also be defined in terms of the ratio of available signal-to-noise power ratio to the available signal-to-noise power ratio at the output

14.8.2 Noise Figure of a Lossy Two-Port Network

This is an important case, where the two-port network considered earlier, is a lossy passive component such as an attenuator or a lossy transmission line, as shown in Figure 14.10.

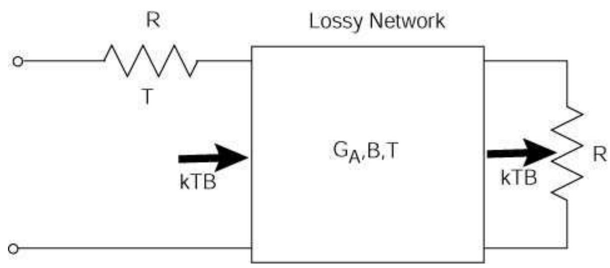


FIGURE 14.10 A lossy two-port network.

A lossy network has a gain ($G_A = \frac{P_O}{P_i}$) less than unity which can be expressed in terms of the loss factor or attenuation (L) as:

$$G_A = \frac{1}{L} \qquad (G_A<1) \qquad (14.27)$$

Since the gain of a lossy network is less than unity it follows that the loss or attenuation factor (L) is more than unity (i.e., $L=P_i/P_o>1$) for any lossy network or component.

Expressing the attenuation factor (L) in "dB" gives the following:

$$L(dB) = -10\log_{10}\left(\frac{P_i}{P_o}\right) \qquad (14.28)$$

For example, if the lossy component attenuates the input power by ten times, then we can write:

$$\mathbf{G_A = \frac{P_O}{P_i} = 0.1 \quad \Rightarrow L=1/G_A=10=10\ dB}$$

If the lossy network is held at a temperature (T), the total available output noise power according to Equation (14.5) is given by:

$$\mathbf{P_{NO}=kTB} \qquad (14.29)$$

On the other hand, from (14.23) the available output noise power is also given by the addition of the input noise power and the generated noise inside the circuit (P_n):

$$\mathbf{P_{NO}=G_AkTB + P_n=KTB/L+P_n} \qquad (14.30)$$

where P_n is the noise generated inside the two-port. Equating Equations (14.29) and (14.30), we obtain P_n as:

$$\mathbf{P_n = \left(\frac{L-1}{L}\right)kTB} \qquad (14.31a)$$

NOTE: *If we refer the noise generated inside the amplifier (P_n) to the input side $(P_n)_i$, from (14.31a) we have:*

$$\mathbf{(P_n)_i=P_n/G_A=LP_n=(L-1)kTB} \qquad (14.31b)$$

Using Equations (14.31) we can now define the equivalent noise temperature (T_e) of a lossy two-port, referred to the input terminals, as:

$$\mathbf{T_e = \frac{(p_n)_i}{kB} \quad \Rightarrow T_e=(L-1)T} \qquad (14.32)$$

Thus the noise figure of a lossy network is given by:

$$\mathbf{F = 1 + \frac{T_e}{T_O}}$$

$$\mathbf{= 1 + (L-1)\frac{T}{T_O}} \qquad (14.33)$$

A SPECIAL CASE:

For a lossy network at room temperature, i.e., $T=T_o$, Equation (14.33) gives:

$$\mathbf{F=L} \qquad (14.34)$$

Equation (14.34) indicates that the noise figure of a lossy network at room temperature equals the attenuation factor (L). For example: if $G_A=1/5$ then $L=1/G_A=5$, giving F=5 or 7 dB for $T=T_o=290$ K.

Example 14.3

A wideband amplifier (2-4 GHz) has a gain of 10 dB, an output power of 10 dBm and a noise figure of 4dB at room temperature. Find the output noise power in dBm.

Solution:

B=2 GHz

GA=10 dB
F=4 dB
$F=P_{NO}/G_AP_{Ni}=P_{NO}/G_AkT_oB$
Thus:
$P_{NO}=FG_AkT_oB$
$10\ \log_{10}P_{NO}=P_{NO}(dB)=F(dB)+G_A(dB)+10\ \log_{10}(kT_o)+10\ \log_{10}(B)$
$=4+10-174+10\ \log_{10}(2x10^9)= -67$ dBm

14.9 NOISE FIGURE OF CASCADED NETWORKS

A microwave system usually consists of several stages or networks connected in cascade where each adds noise to the system thus degrading the overall signal-to-noise ratio.
If the noise figure (or noise temperature) of each stage is known, the overall noise figure (or noise temperature) can be determined.

14.9.1 Cascade of Two Stages

To analyze a two stage amplifier, let us consider a cascade of two amplifiers each with its own gain, noise temperature or noise figure as shown in Figure 14.11. The noise power of each stage is given as follows:

$\mathbf{P_{NO1}=G_{A1}kB(T_o+T_{e1})}$ (14.35)
$\mathbf{P_{NO2}=G_{A2}P_{NO1}+G_{A2}kT_{e2}B}$ (14.36)

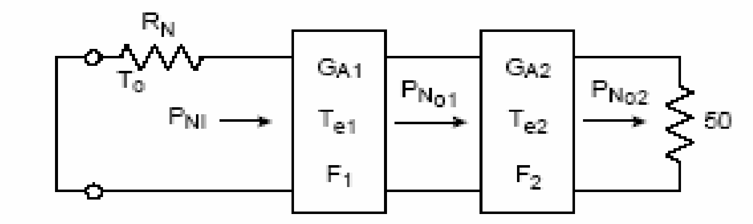


FIGURE 14.11 Cascade of two stages.

Combining Equations (14.35) and (14.36) we get:
$\mathbf{P_{NO2}=G_{A1}G_{A2}kB(T_o+T_{e1}+T_{e2}/G_{A1})}$ (14.37)
The two-stage amplifier as a whole has an total gain of $G_A=G_{A1}G_{A2}$, an overall equivalent noise temperature (T_e) and a total output noise power (P_{NO}) given by:
$\mathbf{P_{NO}=G_AkB(T_O+T_e)}$ (14.38)
Comparing Equation (14.38) to (14.37) we have:
$\mathbf{T_e=T_{e1}+T_{e2}/G_{A1}}$ (14.39)

The overall noise figure (F) for the two-stage amplifier is found by using (14.39):
$\mathbf{F=1+T_e/T_o=1+(T_{e1}+T_{e2}/G_{A1})/T_o}$ (14.40)
By noting that:
$\mathbf{F_1=1+T_{e1}/T_o,}$ (14.41)
$\mathbf{F_2=1+T_{e2}/T_o}$ (14.42)
Equation (14.40) can be written as:

$$\mathbf{F = F_1 + \frac{F_2 - 1}{G_{A1}}} \quad (14.43)$$

Equations (14.39) and (14.43) show that the first stage noise figure F_1 (or noise temperature, T_{e1}), and gain (G_{A1}) have a large influence on the overall noise figure (or noise temperature). This is because the second stage noise figure, F_2 (or noise temperature, T_{e2}) is reduced by the gain of the first stage (G_{A1}).

Thus the key to low overall noise figure is a primary focus on the first stage by reducing its noise and increasing its gain. Later stages have a greatly reduced effect on the overall noise figure.

NOISE MEASURE

In order to determine systematically the order or sequence in which two similar amplifiers need be connected to produce the lowest possible noise figure, we first must define a quantity called "noise measure" as:

$$\mathbf{M = \frac{F - 1}{1 - 1/G_A}} \qquad (14.44)$$

If amplifier #1 (AMP1) has a noise measure (M_1) and amplifiers #2 (AMP2) a noise measure (M_2) then there are two possible cases which needs to be addressed (in order to obtain the lowest possible noise form the cascade), as follows:

Case I: $M_1<M_2$ -- Then AMP1 should precede AMP2 since $F_{12}<F_{21}$

Case II: $M_2<M_1$ -- Then AMP2 should precede AMP1 since $F_{21}<F_{12}$

Where F_{12} is the overall noise figure of the two stage amplifier when AMP1 precedes AMP2; and vice versa F_{21} is for the case when AMP2 precedes AMP1.

NOTE: *It can easily be shown mathematically that for example:*

If $\boldsymbol{M_1<M_2}$ ***then*** $\boldsymbol{F_{12}<F_{21}}$ (14.45)

where

$$\mathbf{F_{12} = F_1 + \frac{F_2 - 1}{G_{A1}}} \qquad (14.46)$$

$$\mathbf{F_{21} = F_2 + \frac{F_1 - 1}{G_{A2}}} \qquad (14.47)$$

And vice versa, if $\boldsymbol{M_2<M_1}$ ***then*** $\boldsymbol{F_{21}<F_{12}}$.

Example 14.4

An antenna is connected to an amplifier via a transmission line which has an attenuation of 3 dB (see Figure 14.12). The amplifier has the following specifications:

G_A*=20 dB*

B=200 MHz

T_e*=145 K*

Calculate the overall noise figure and gain of the cascade at 300 K.

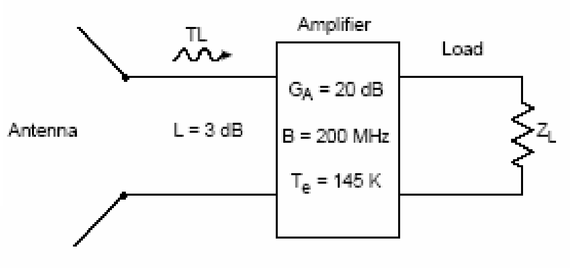


FIGURE 14.12 Circuit for Example 14.4.

Solution:
a. For the transmission line we have:
Since $L=1/G_{TL} \Rightarrow L(dB)=-G_{TL}(dB)$
$L=3\ dB=2 \ \Rightarrow G_{TL}=-3\ dB=1/2$
$F_{TL}=1+(L-1)T/T_o=1+(2-1)300/290=2.03=3.1\ dB$
b. For the amplifier we have:
$F_{AMP}=1+T_e/T_o=1.5=1.8\ dB$
c. The overall noise figure and gain are calculated to be:
$F_{TOT}=F_{TL}+(F_{AMP}-1)/G_{TL}=2.03+(1.5-1)/0.5=3.03=4.8\ dB$
$G_{TOT}=G_{TL}+G_{AMP}=-3+20=17\ dB$

Therefore we can see that due to the addition of a lossy transmission line in front of the amplifier, we have three deleterious effects: 1) the overall noise figure has increased (from 1.8 dB to 4.2 dB) 2) the second stage noise contribution has been intensified since the transmission line has a gain less than unity ($G_{TL}<1$), and 3) the overall gain dropped by 3 dB which represents the third side effect.

14.9.2 Cascade of n Stages

For a cascade of "n" amplifiers (see Figure 4.13), the overall noise figure is the generalization of equations for equivalent noise temperature ($T_{e,cas}$) and noise figure (F_{cas}) of a two-stage cascade as follows:

$$\mathbf{T_{e,cas} = T_{e1} + \frac{T_{e2}}{G_{A1}} + \frac{T_{e3}}{G_{A1}G_{A2}} + \cdots + \frac{T_{en}}{G_{A1}G_{A2}\cdots G_{An-1}}} \qquad (14.48a)$$

$$\mathbf{F_{cas} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1}G_{A2}} + \cdots + \frac{F_n - 1}{G_{A1}G_{A2}\cdots G_{An-1}}} \qquad (14.48b)$$

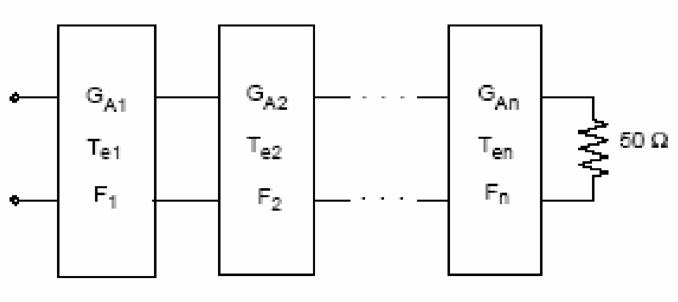


FIGURE 14.13 Cascade of n amplifier stages.

SPECIAL CASE: IDENTICAL STAGES

If all stages are identical, i.e.,

$$G_{A1}=G_{A2}=....=G_{An}=G_A \quad (14.49a)$$

$$T_{e1}=T_{e2}=.....=T_{en}=T_e \quad (14.49b)$$

$$F_1=F_2=......=F_n=F \quad (14.49c)$$

Then Equations (14.48a,b) would greatly simplify as follows:

$$T_{e,cas}=T_e(1+X+X^2+....+X^{n-1}), \quad (14.50a)$$

$$F_{cas}=(F-1)(1+X+X^2+....+X^{n-1})+1 \quad (14.50b)$$

Where

$$X=\frac{1}{G_A}$$

Using the following identity for the geometric series:

$$1+X+X^2+.....+X^{n-1} = (1-X^n)/(1-X), \quad |X|<1 \quad (14.51)$$

We can write Equations (14.50a,b) as:

$$T_{e,cas}=T_e\left(\frac{1-(1/G_A)^n}{1-1/G_A}\right) \quad (14.52a)$$

$$F_{cas}=(F-1)\left(\frac{1-(1/G_A)^n}{1-1/G_A}\right)+1 \quad (14.52b)$$

AN INFINITE CHAIN OF IDENTICAL AMPLIFIERS

If n is very large (i.e., $n\rightarrow\infty$) then:

$$\text{Lim}_{n\rightarrow\infty}(X)^n=0, \quad |X|<1 \quad (14.53a)$$

And the geometric series identity in Equation (14.51) further simplifies into:

$$1+X+X^2+.....+X^{n-1}+....=\frac{1}{1-X}, \quad |X|<1 \quad (14.53b)$$

Using Equation (14.53b), we can see that Equations (14.52a,b) and for an infinite chain of amplifiers become:

$$T_{e,cas}=T_e\left(\frac{1}{1-1/G_A}\right) \quad (14.54a)$$

$$\mathbf{F_{cas} = (F-1)\left(\frac{1}{1-1/G_A}\right)+1} \quad (14.54b)$$

In terms of "noise measure", M, defined earlier as:

$$\mathbf{M = \frac{F-1}{1-1/G_A}} \quad (14.55)$$

we can write (14.54) as:

$$\mathbf{T_{e,cas} = T_e\left(\frac{M}{F-1}\right)} \quad (14.56a)$$

$$\mathbf{F_{cas} = M+1} \quad (14.56b)$$

NOTE 1: *For a "Minimum-noise amplifier", where each stage operates at minimum noise figure (i.e., F_1=F_2=....=F_n=F_{min}), we have:*

$$\mathbf{M_{min} = \left(\frac{F_{min}-1}{1-1/G_A}\right)} \quad (14.57)$$

We can write (14.56) as:

$$\mathbf{T_{e,cas} = T_{e,min}\left(\frac{M_{min}}{F_{min}-1}\right)} \quad (14.58a)$$

$$\mathbf{F_{cas} = M_{min}+1} \quad (14.58b)$$

NOTE 2: *If the gain of each stage is very large (i.e., $G_A \rightarrow \infty$), then Equation (14.56) becomes:*

$\mathbf{G_A \rightarrow \infty \Rightarrow M=F-1}$ (14.59)

$\mathbf{T_{e,cas}=T_e}$ (14.60a)

$\mathbf{F_{cas}=F}$ (14.60b)

This result indicates that a large cascade of very high-gain amplifiers will only result in the degradation of the signal by the first stage and the effect of all the many stages is null and void as far as the added noise is concerned.

This result is in agreement with the conclusion made earlier, in which it became apparent that the first stage's gain and noise figure value dominates and greatly affects the overall noise figure of the cascade.

Chapter 14- Symbol List

A symbol will not be repeated again, once it has been identified and defined in an earlier chapter, with its definition remaining unchanged.

B - Bandwidth

F - Noise Figure

k - Boltzmann’s constant

M - Noise measure
N - Overall noise Figure
P_N - Noise power
P_{NI} - Input Noise power
$P_{N0,tot}$ - Total output noise power
R_N - Noisy resistor
R_{N0} - Noiseless resistor
T - Temperature
T_e - Equivalent noise temperature
T_0 - Standard room temperature (290° K)
T_S - Source and load temperature
$V_{n,rms}$- Root-mean-square (rms) of noise
$\overline{V_n^2}$ - The mean-square value of noise

CHAPTER-14 PROBLEMS

14.1) The Y-factor method is to be used to measure the equivalent noise temperature of a component. A hot load of T_1=300 K and a cold load of T_2=77 K will be used. If the noise temperature of the amplifier is T_e=250 K, what will be the ratio of power meter readings at the output of the component for the two loads?

14.2) A transmission line has a noise figures F=1 dB at a temperature T_O=290 K. Calculate and plot the noise figure of this line as its physical temperature ranges from T=0 K to 1000 K.

14.3) Assume that measurement error introduces an uncertainty of ΔY into the measurement of Y in a Y-factor measurement. Derive an expression for the normalized error of the equivalent noise temperature ($\Delta T_e/T_e$) in terms of ($\Delta Y/Y$) and the temperatures T_1, T_2 and T_e. Plot ($\Delta T_e/T_e$) as a function T_e for two values of ($\Delta Y/Y$): a) 0.1, and b) 0.20, and from these plots establish the minimum normalized error ($\Delta T_e/T_e$) and the corresponding T_e for each case.

14.4) An amplifier with a bandwidth of 1 GHz has a gain of 15 dB and a noise temperature of 250 K. If it is used as a preamplifier in a cascade, preceding a microwave amplifier of 20 dB gain 5 dB noise figure, determine the overall noise temperature.

14.5) An amplifier with a gain of 12 dB, a bandwidth of 150 MHz and a noise figure of 4 dB feeds a receiver with a noise temperature of 900K. Find the noise figure of the overall system.

14.6) Consider the microwave system shown in Figure P14.6, where the bandwidth is 1 GHz centered at 20 GHz and the physical temperature of the system is T_O=300 K. What is the equivalent noise temperature of the source? What is the noise input of the amplifier in dB? When the noisy source is connected to the system what is the total noise power output of the amplifier in dBm?

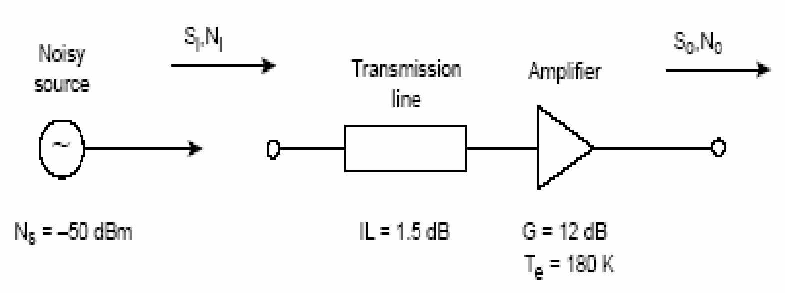


FIGURE P14.6

14.7) Consider the wireless local area network (WLAN) receiver front-end shown in Figure P14.7, where the bandwidth of the bandpass filter is 100 MHz centered at 2.4 GHz. If the system is at room temperature, find the noise figure of the overall system. What is the resulting signal to noise ratio at the output if the input signal power level is -90 dBm? Can the components be rearranged to give a better noise

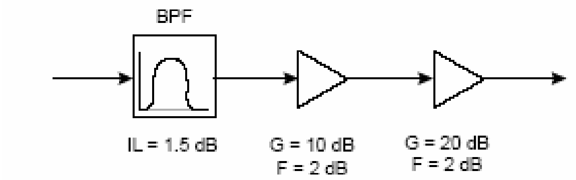


FIGURE P14.7

figure?

14.8) A two-way power divider has one output port terminated in a matched load as shown in Figure P14.8. Find the equivalent noise temperature of the resulting two-Port network if the divider is an equal-split two-way resistive divider.

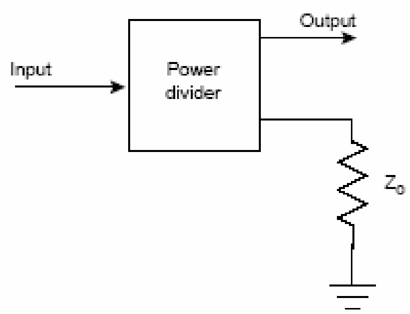


FIGURE P14.8

14.9) For a two-stage cascaded network with gain values of G_1 and G_2 and noise figures of F_1 and F_2 as shown in Figure P14.9, the input noise power is N_i=kTB. The output noise power is N_1 and N_2 at the output of the first and second stages. Are the following expressions correct:

a. $F_1=N_1/G_1N_i$

b. $F_2=N_2/G_2N_1$

c. $F_2=N_2/G_1G_2F_2N_i$

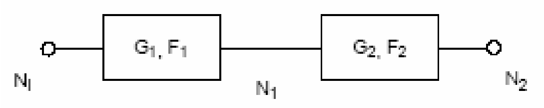


FIGURE P14.9

14.10) A receiver has the block diagram shown in Figure P14.10. Calculate:
a. The total gain (or loss) in dB,
b. The overall noise Figure in dB.

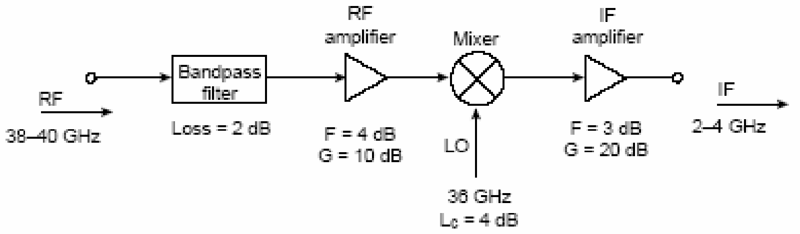


FIGURE P14.10

14.11) Two satellite receiver systems have the following specifications for their components:
RF Amplifier: F=5 dB, G=10 dB
Mixer: L_c=5 dB
IF amplifier: F=2 dB, G=15 dB
Bandpass filter: IL=2 dB
Compare the two systems in terms of the overall gain and noise figure values (see Figure P14.11).

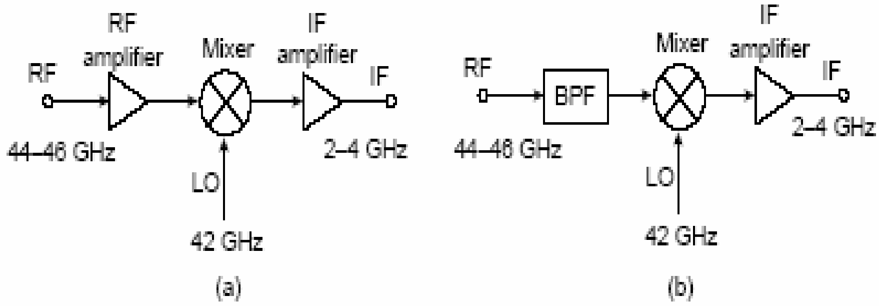


FIGURE P14.11

14.12) Calculate the overall noise Figure and gain for the receiver system shown in Figure P14.12.

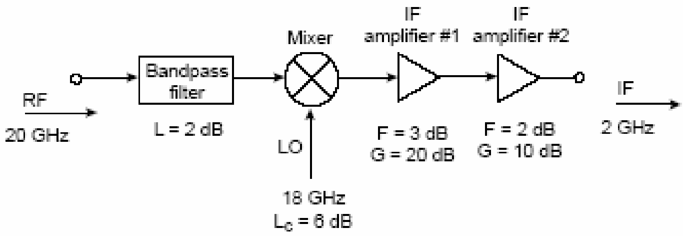


FIGURE P14.12

14.13) The S parameters and the noise parameters of a GaAs FET at 10 GHz are:

$$\mathbf{S} = \begin{bmatrix} \mathbf{0.6\angle -170^\circ} & \mathbf{0.05\angle 16^\circ} \\ \mathbf{2\angle 30^\circ} & \mathbf{0.5\angle -95^\circ} \end{bmatrix}$$

$\mathbf{F_{min}=2.5\ dB}$

$\mathbf{\Gamma_O=0.5\angle 145^\circ}$

$R_n = 5\ \Omega$

a. Is the transistor unconditionally stable?

b. Determine $G_{A,max}$

c. Determine the noise figure if the transistor is used in an amplifier designed for maximum available gain ($G_{A,max}$)

14. 14) Consider the low noise block (LNB) shown in Figure P14.14. Calculate the total noise figure and the available gain of this block.

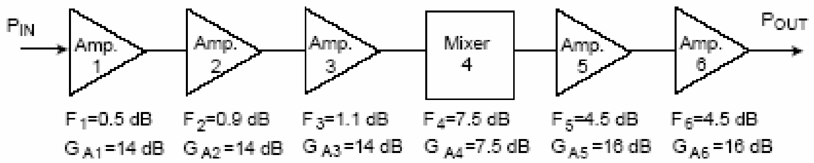


FIGURE P14.14