Part 1: Introduction to Crystal Properties

Semiconductors

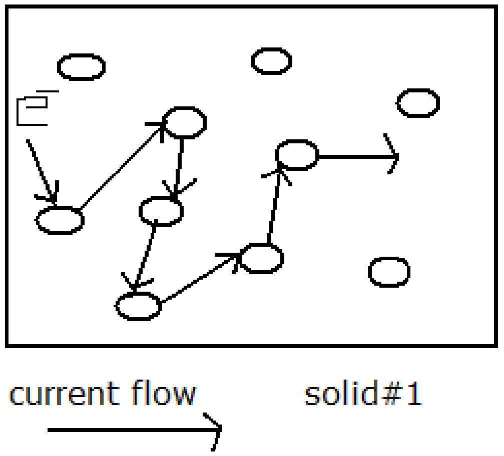
- **Solid materials**
- **Semiconductor Latices**
- **Miller Indices**

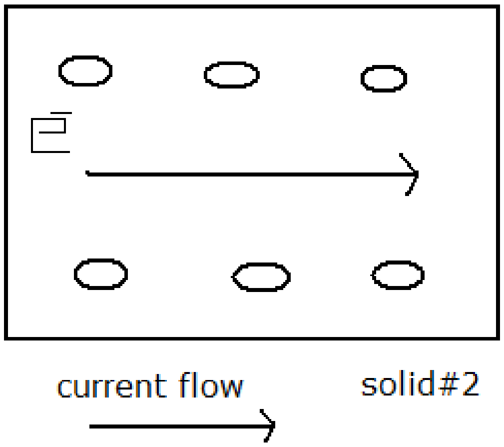
1.1 INTRODUCTION TO CRYSTAL PROPERTIES

In studying solid state electronic devices, we are interested in the electrical behavior of solids. Charge transport depends on:

a) Properties of electrons (e^-)

b) Arrangement of atoms in the solid





Some basic properties of semiconductors compared with other solids are presented in this section. This section also covers crystal structures of semiconductor as well as types of semiconductors.

Solid state materials are divided into:

a) Insulators

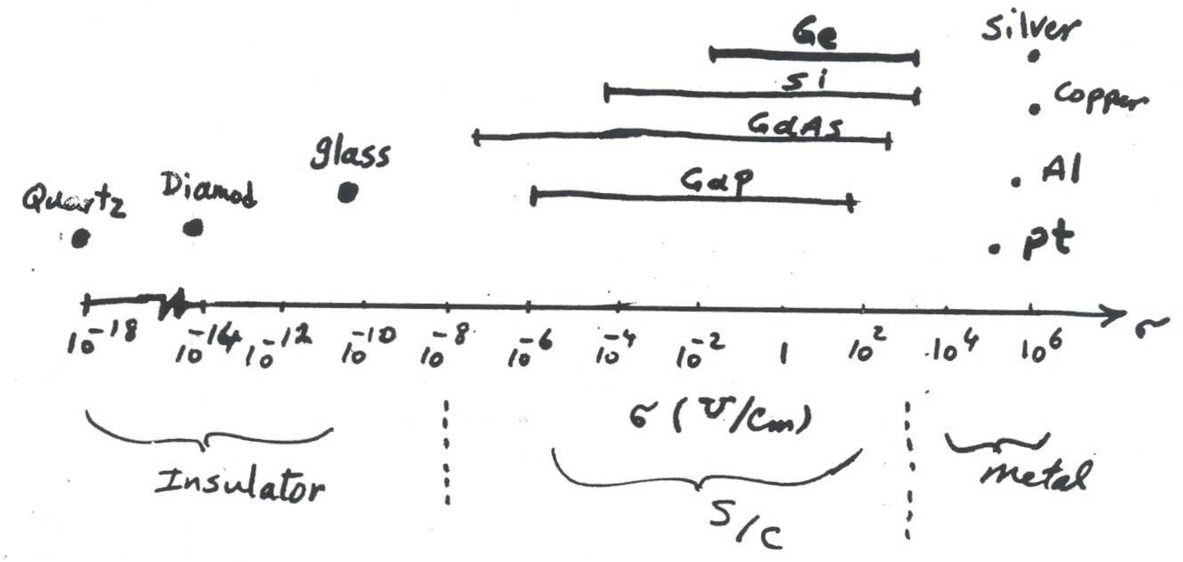
b) Semiconductors

c) Metals

Semiconductors are a group of materials with conductivity which lie between insulators and metals.

Insulators (10^{-8}) < σ < metals(10^{3})

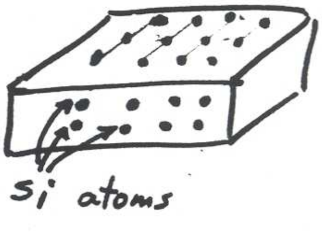
where σ = electrical conductivity



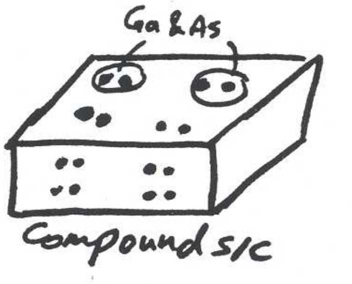
SEMICONDUCTORS

Semiconductors are divided into two types:

a) Elemental: Made up of single species of atoms. e.g. Si,Ge (column IV of the periodic table)



b) Inter metallic or compound: Made up of atoms belonging to column III & V and II & VI



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Periodic Table

II	III	IV	V	VI
Zn	B	C	P	S
Cd	Al	Si	As	Se
	Ga	Ge	Sb	Te
	In			

Elemental	IV compound	III-V compound	II-IV compound
Si	Si C	Ga As	Cd Te
C	Si Ge	Ga P	Cd Se
		In As	Zn S
		In P	Zn Se
		Al As	Zn Te
		Al P	Cd S

Applications:

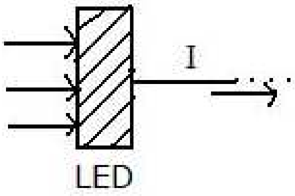
a) Ge: Was popular for x tors and diodes in earlier days

b) Si: Is now used for x tors, diodes and ICs

c) Compound semiconductors: Are widely used in high speed devices and devices requiring emission or absorption of light.

Examples: 1) Ga As -(binary III-V) used in microwave IC (MMIC)

2) Ga As,Ga P – used in LEDs, photonic devices



3) Ga As(ternary compound) and In Ga As p (quaternary compound)

these add extra flexibility

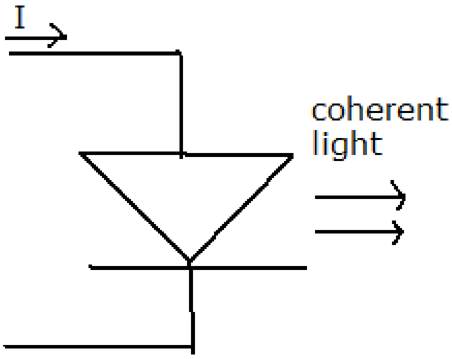
4) Zn S(binary II-VI) -used in TV screens

5) Pb Te,Hg Cd Te,In Sb,Cd Se: Used in light detectors

6) Si&Ge: Used in Infra-red and nuclear radiator detectors

7) Ga As,In P: Used in microwave devices e.g. Gunn diode

8) Ga As,Al Ga As: Used in semiconductor lasers

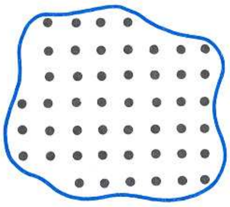


Laser
opposite of LED

SOLID MATERIALS

Solid materials classified by atomic arrangement:

1) Crystalline Solid or Crystal:



(a) Crystalline

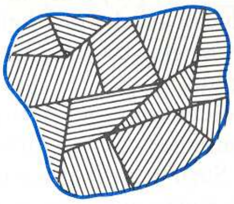
Atoms making up the crystal are arranged in a periodic fashion.

The periodic arrangement of atoms in a crystal is called the **“ lattice “.**

A **“unit cell”** is a volume which is representative of the entire lattice and is regularly repeated throughout the crystal.

Example: Lam-grown silicon

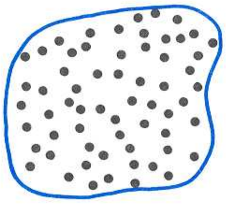
2) Polycrystalline Solid: Is composed of many small regions of single crystal material.
Example: Randomly cooled purified Si or Ge.



(c) Polycrystalline

3) Amorphous Solid: Is composed of atoms having no periodic structure at all. It has no lattice.

Example: SiO_2 (glass).



(b) Amorphous

Some Basic Unit Cells

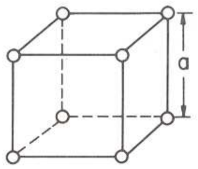
1) Simple cubic crystal (SSC)

a = Lattice Constant (in cubic unit cells)

Ex: Polonium

Number of atoms=1/unit cell

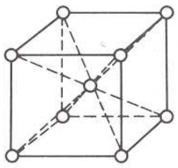
Each atom has six nearest neighbors.



SIMPLE CUBIC
(P, etc)

2) **Body-Centered Cubic Crystal (BCC):**

Ex: Sodium, Tungsten



BODY-CENTERED CUBIC
(Na, W, etc)

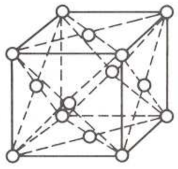
Number of atoms=2/unit cell

Each atom has eight nearest neighbors (eight corners of cube)

3) **Face-Centered Cubic Crystal (FCC):**

It has one atom at each of the six cubic faces in addition to the eight corners of atom.

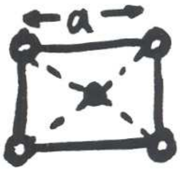
Number of atoms=1(cubic)+3(faces)=4/unit cell

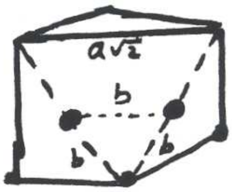


FACE-
CENTERED CUBIC
(Aℓ, Au, etc)

Each atom has 12 nearest neighbors.

Why? -Each face centered atom has 4 neighbors at the corners.





Each face centered atom has 4 face centered nearest neighbors at the top half.

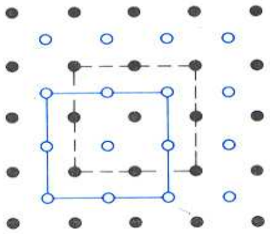
Each face atom has 4 face centered nearest neighbors at the bottom half.

$$b = \frac{a\sqrt{2}}{2}$$

Ex: Al,Cu,Pt,Au

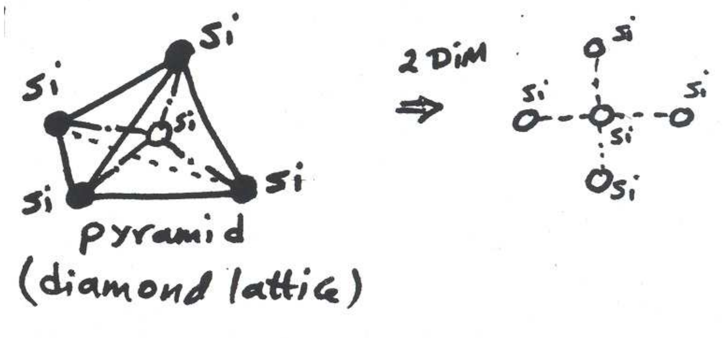
Semiconductor Lattices

1) The elemental semiconductor (Si & Ge) has a diamond lattice structure. This structure also belongs to the cubic crystal family and can be seen as “two inter penetrating FCC sub lattices” displaced from each other (a/4,a/4,a/4) .i.e. a displacement of $a\sqrt{3}/4$. (diagonal of the cube = $a\sqrt{3}$).



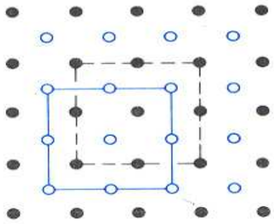
A 2-Dimensional view is shown above.

Each atom is surrounded by 4 equidistant nearest neighbors that lie at the corners of a Tetrahedron (a solid figure with 4 faces) forming a pyramid (i.e. all sides equal).



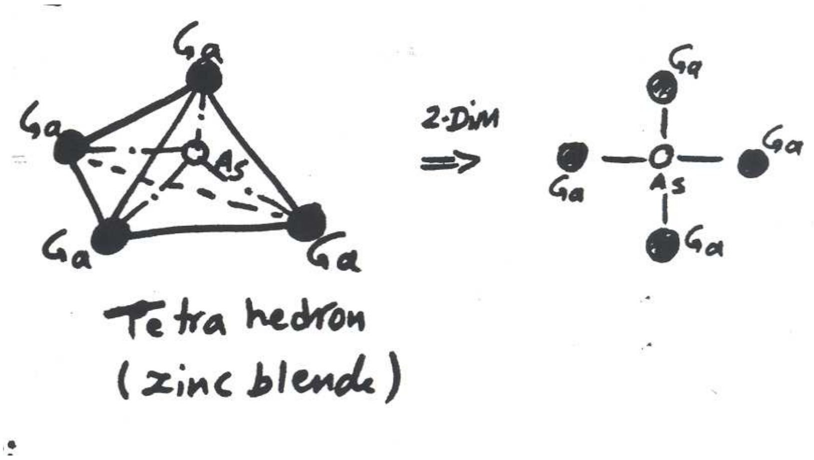
2) **Zinc Blend Lattice:**

Most of the III-V_{th} compound semiconductors (e.g. As.Etc) have a Zincblende lattice which is identical to a diamond lattice except that one FCC sub lattice has column III atoms (Ga) and the other has column V atoms (As)



A 2-Dimension view is shown above.

The 4 nearest neighboring atoms would lie at the corner of a tetrahedron with Ga & As. Each having different atomic radii.



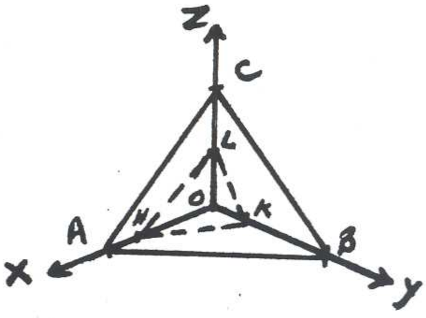
Miller Indices

Crystal properties are different along different planes, thus electrical and other devices characteristics are dependent on xtal orientation. A convenient method of defining the various planes in an xtal is to use “**miller indices**” which are obtained using the following steps:

1. Find the intercepts of the plane on the three Cartesian coordinates in terms of lattice constant (a)
2. Take the reciprocal of these numbers (this avoids infinity $^{\infty}$)
3. Reduce numbers (of step 2) to the smallest 3 integers having the same ratio.

ABC = Unit Plane

HKL = a xtal plane



$\left(\frac{OA}{OH} , \frac{OB}{OK} , \frac{OC}{OL} \right) \rightarrow (h,k,l)$

Ex: 1) Intercepts OH = 2a

OK = 4a

OL = a

2) (1/2 , $\frac{1}{4}$, 1) ← (reciprocal)

3) (2, 1, 4) ←(multiply by 4) ==>miller indices = (2,1,4)

This is (2 1 4) plane!

Note 1: Each set of numbers define a set of parallel planes in the lattice.

Note 2: If a plane passes through the origin, it can be translated to a parallel position for calculation of miller indices.

Note 3: If an intercept occur on the negative part of an axis, the minus sign is placed above the index I.e. (h k l)

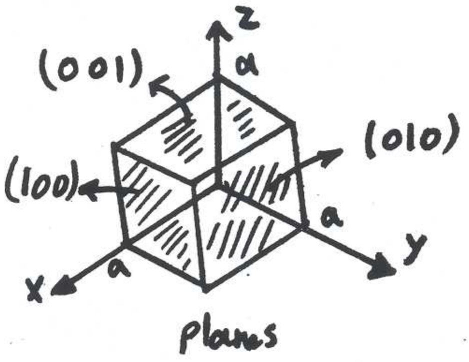
Note 4: For equivalent planes use {h k l}

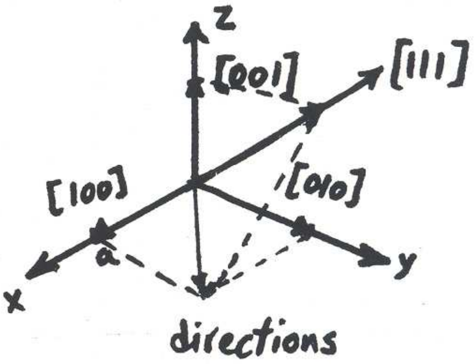
Ex : {100}= (100),(010),(001),(T00),(0T0)&(00T)

Note 5: For a direction in a lattice a set of three integers expressed as the multiplies of lattice const. is used : [h k l]

Note 6: For a full set of equivalent direction use < h k l >

Ex: <100> = [100],[010],[001],[100],[010]&[001]





Note : dir.[hkl] is perpendicular to plane (hkl) --->true in cubic system!

PART 2

Energy bands and charge carriers: Basic Principles

Chapter 1: Energy bands and charge carriers

Chapter 2: Excess carriers in semiconductor

Chapter 1: Energy Bands and Charge Carriers

A specific mechanism by which electric current flows in a solid is examined. We shall learn about good and poor conductors of electric current as well as examining what makes these conductors good or bad. We will also study variation of semiconductor conductivity by changing the temperature or the number of impurities.

1.1 – Bonding forces and energy bands

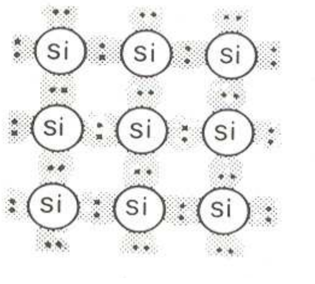
we will discuss bonding forces and energy bands

Interaction of electrons (e^-) in neighboringin this section.

1.1.1 Bonding forces

atoms of a solid holds the crystal together. There are three types of atomic interaction in a solid which leads to three types of bonding in crystals:

a. Ionic Bonding: The crystal is made up of ions with the electronic structure of inert atoms, but the ions have net electric charges after electron exchange.



Note: All electrons are tightly bound to atoms .No free electronics .i.e. good insulators.

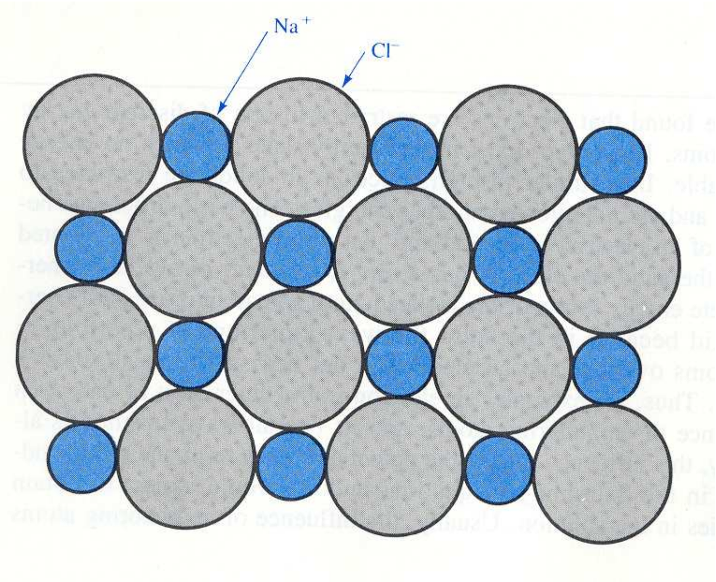
b. Metallic Bonding: In a metal, the outer electronic shell is only partially filled (usually

less than or equal to $3e^-$). These electrons are loosely bound and is given up easily to the crystal as

a whole, so that the solid is made up of ions with closed shells immersed in a sea of free electrons.

These electrons are free to move about the lattice under an E-field influence.

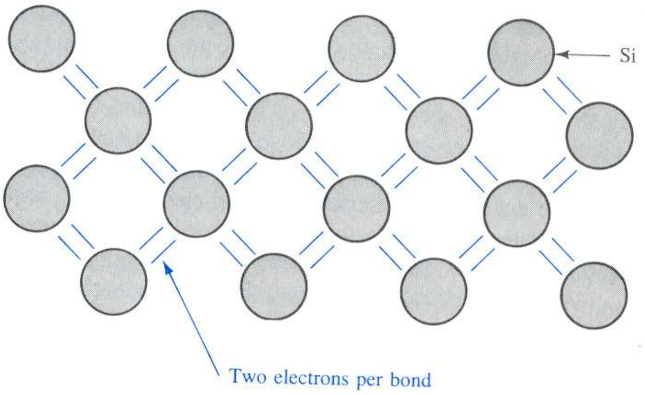
E.g. Na metal



Na metal

c. Covalent Bonding: In this type of atomic interactions each atom shared its valence electrons with its four neighbors. Each electron-pair constitute a covalent bond.

E.g. Ge,Si,C { column IV }



Shared electrons are tightly bound to the lattice (except at T=0^{o}k) and can be thermally or optically excited out of a covalent bond and thereby becoming free and increasing conductivity (semiconductors).

1.1.2 Energy Bands

FACT: Electrons in an atom are restricted to sets of discrete energy levels. Large gaps exist in the energy scale in which no energy states are available.

As isolated atoms are brought together to form a solid, various atomic interactions (3 types of bonding) occur between neighboring atoms. The forces of attractions and repulsion between atoms will find a balance at the proper inter-atomic spacing for the crystal.

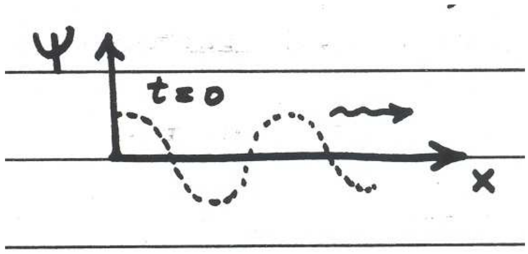
Similarity: Electrons in solids are restricted to certain energies and are not allowed at other energies.

Difference: Electrons in a solid have a range or band of available energies and are not at discrete energy levels of an atom. In a solid the discrete energy levels of isolated atoms spread into band of energies. These bands form practically continuous levels of energies.

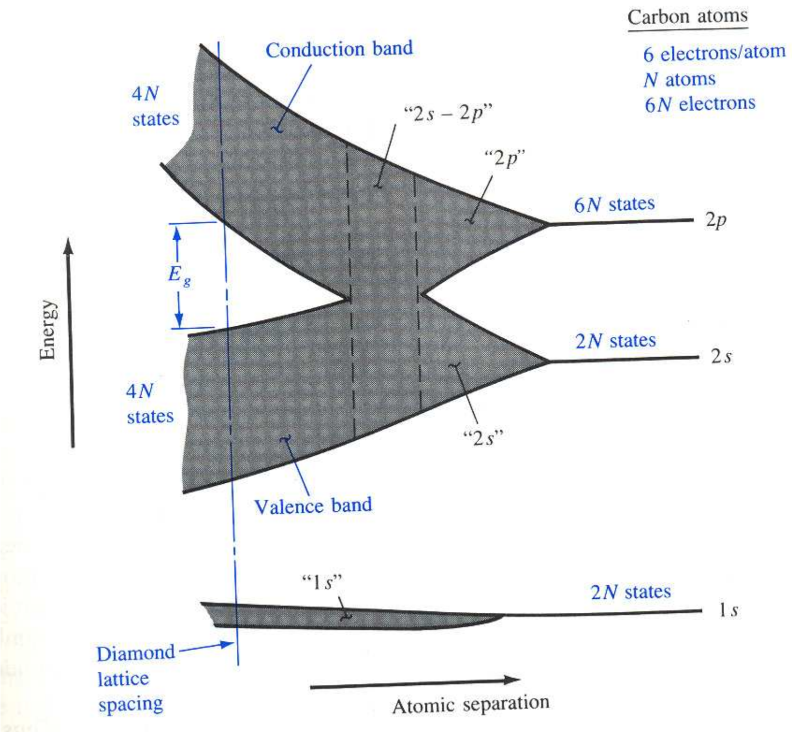
Reason for band of energies: Because in the solid the wave function of electrons in neighboring atoms overlap and an electron is not necessarily localized at a particular atom, i.e. Electrons are shared between atoms and are mobile.

Note: According to Quantum Theory, an electron can be considered a wave as well as a particle. Wave function of an electron is the amplitude and phase of its motion as a wave.

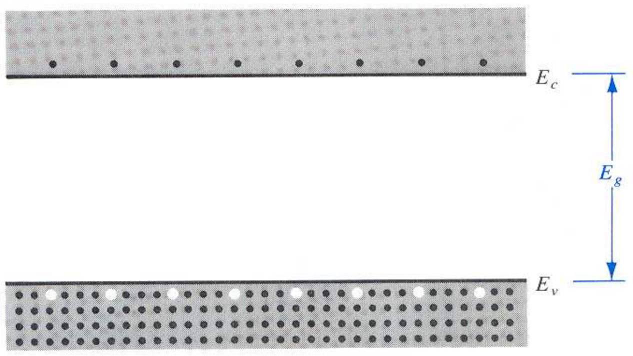
E.g. $\psi = a \cos(wt-kt)$



Conclusion: Influence of neighboring atoms on the energy levels of a particular atom causes the shifting and splitting of energy states into energy bands. E.g. Carbon atom ($1s^2\,2s^2\,2p^2$)



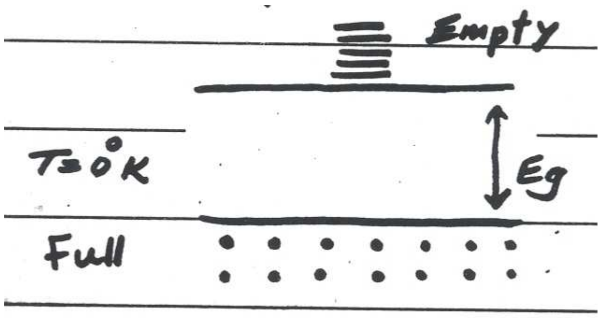
Note: As the distance between atoms approached the equilibrium inter-atomic spacing, this band splits into two bands separated by an energy gap(E_g)



Electron count: 1s band: 2N e^-

valence band: $4Ne^-$ (at 0^o K)

Conduction band: 0

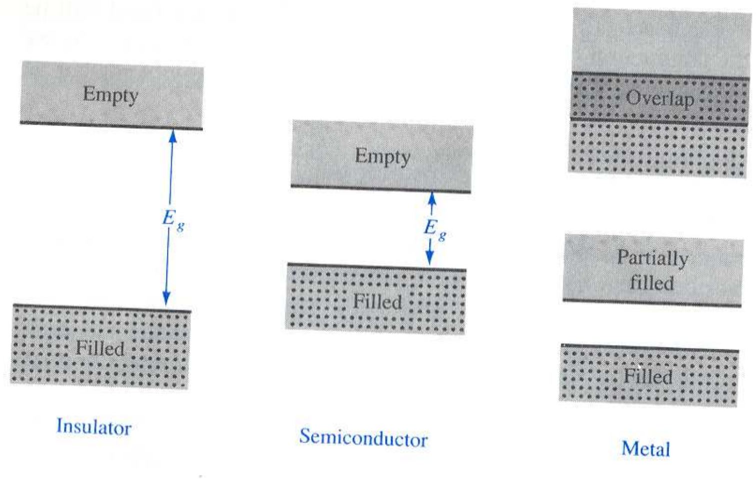


1.2 Charge Carriers in Semiconductors

1.2.1 – Metals, Semiconductors , Insulators-Energy bands observation:

Every solid has its own characteristic energy band structure. This variation in band structure causes different electrical properties.

FACT: Electric current flow in solid under an applied E-field depends on free electrons ability to move into new energy states. I.e. There must be empty allowed states available to the electron.



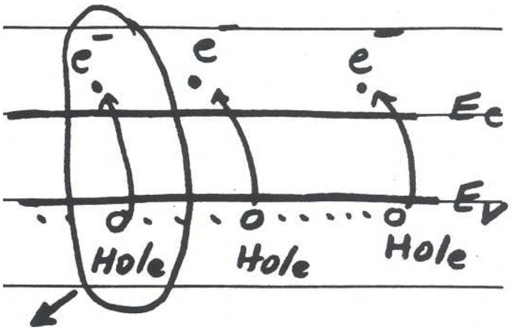
Observation: At room temperature, a semiconductor with 1ev band gap will have a large number of electrons in conduction band due to thermal excitation, unlike an insulator.

1.2.2- Electrons and Holes:

Observation: As the temperature of a semiconductor is raised above 0^o k , some electrons in the valence receive enough thermal energy to be excited across the band gap to the conduction band.

Convention: electron = e^-

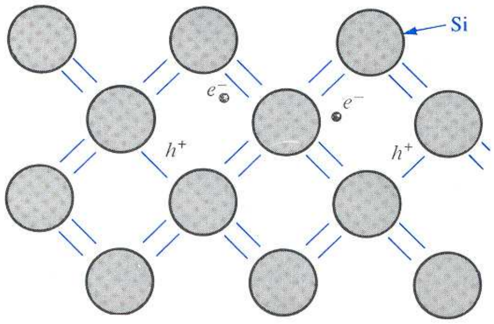
hole = h^+

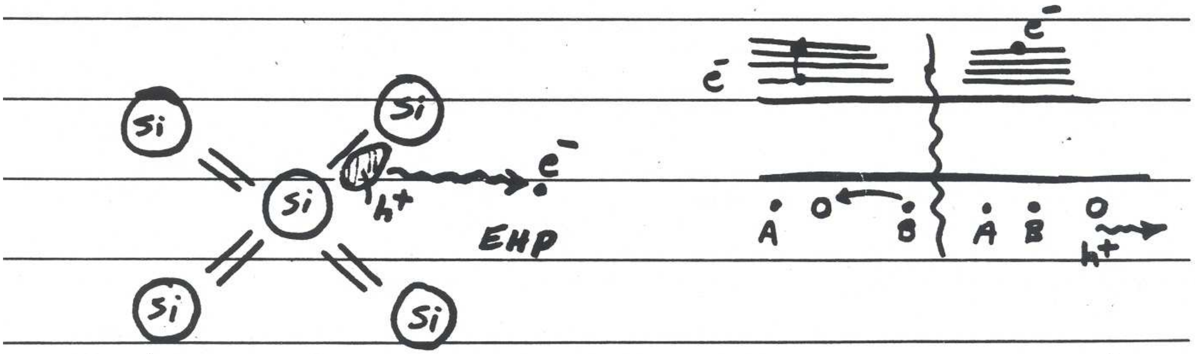


Electron hole pair

EHP is equivalent to a broken bond!

Upon on generation of EHP, an electron in the conduction band is surrounded by a large number of unoccupied states. Similarly, holes in the valiancy band contribute to charge transport by allowing other electrons to hop around by occupying the hole. Thus allowing the hole to move around.





1.2.3- Concept of Effective Mass:

Observation: Electrons in a crystal are not completely free but instead interact with the periodic lattice, thus their motion cannot be expected to be the same as for electrons in free space.

Effective mass: In applying equations of electrodynamics (science of electricity in motion and interaction with magnetic fields), an altered value of particle mass is used, to account for most of the lattice influence. This we call Effective mass (m^*).

Conclusion: Using effective mass(m^*), electrons and holes can be treated as almost free carriers in most computations.

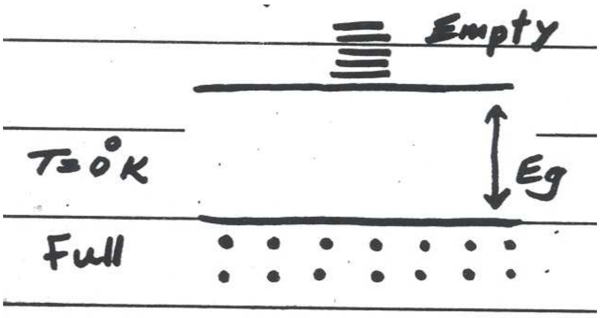
Example: for Si Ga As Ge

$m_n^* = 1.1\ m_o$	$0.067m_o$	$0.55m_o$
$m_p^* = 0.56\ m_o$	$0.48m_o$	$0.37m_o$

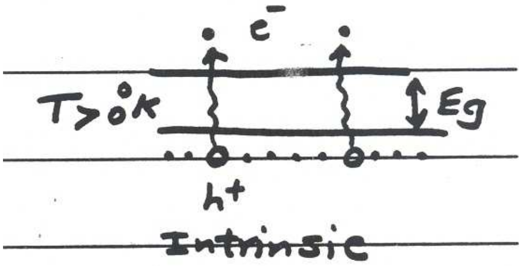
m_o = free electron rest mass.

1.2.4- Intrinsic Semiconductors:

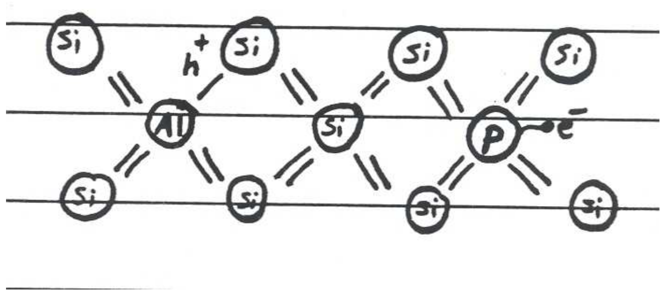
Intrinsic semiconductors: Is a crystal with no impurities or lattice defects. At 0^o k , there are no free electrons in conduction band and valence band is filled with electrons.



Observation: At higher temperatures EHP are generated as valence band electrons are excited thermally across the band gap to the conduction band. These EHPs are the only charge carries in intrinsic material.



Visualization: Generation of EHPs corresponds to the breaking of covalent bonds. The energy to break the bond is the band gap energy E_g.



Observation: Since the electrons and holes are created in pairs the conduction band electrons concentration (n e^-/cm^3) is equal to the concentration of holes in the valence band (p h^+/cm^3), thus

$$n=p=n_i \quad \text{for intrinsic semiconductor}$$

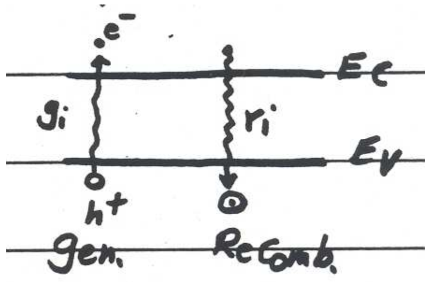
New Terms:

1. **Generation:** Is the creation of an EHP (gi EHP/ cm^3-s)
2. **Recombination:** Occurs when an electron in the conduction band makes a transition to an empty state (hole) in the valence band.(ri=recombination rate EHP/cm^3-)

Observation: For an equilibrium carrier concentration, there must be recombination of EHPs at the same rate at which they are generated. I.e. ri=gi, at equilibrium ri and gi are temperature dependent and are proportional to the equilibrium concentration of electrons and holes.

$$Ri = \alpha_r n_o p_0 = \alpha_r n_i^2 = gi \qquad \text{at any temperature (T)}$$

Where α_r = proportionality constant



1.2.5 Extrinsic Semiconductor:

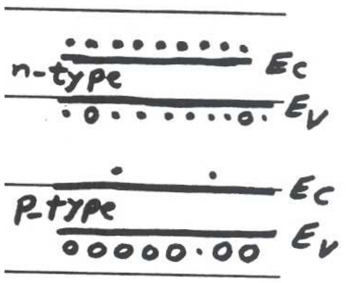
New Term:

1. **Doping:** Is the process of adding the controlled amounts of impurities to a semiconductor crystal in order to control its resistivity.

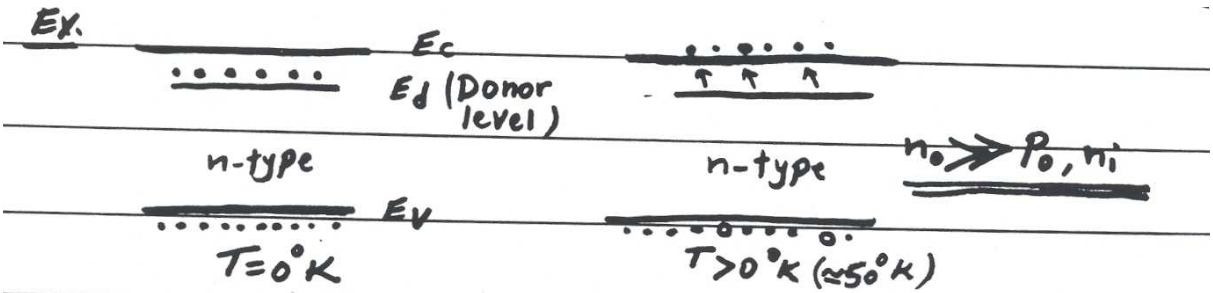
Definition: When a crystal is doped such that is the equal carrier concentration n_o and p_o are different from intrinsic concentration(n_i) ,The material is said to be **Extrinsic**. There are two types of Extrinsic semiconductors, n-type(charge carrier = e^- majority),and p-type(charge carrier = h^+ majority)

In Short: n-type semiconductor: $n_o >> p_o, n_i$

p-type semiconductor : $p_o >> n_o , n_i$

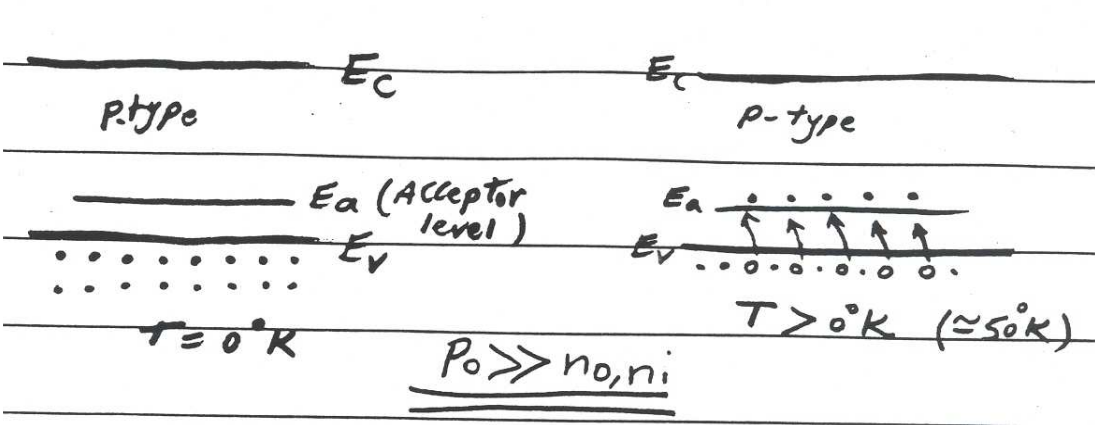


Band Theory: When impurities or lattice defects are introduced into an intrinsic semiconductor, additional levels are created in the energy band structure, usually within band gap.

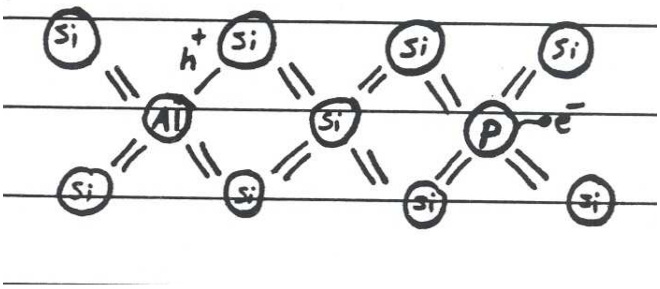


Impurity: Column V (P, As, sb) => n type (as above)

Impurity: column III (B,Al,Ga,In) ➔ p-type



Corresponding covalent bonding model of a semiconductor:



1.3. Carrier Concentration:

The purpose of this section is to obtain equations for carrier concentration, while understanding the distribution of carriers over the available energy states.

Fact: Due to the nature and randomness of motion of electrons the distribution of carriers are meaningful only by a statically method called Fermi-Dirac statics which takes into account the indistinguishable nature of electrons, their wave nature and the Pauli Exclusion Principle.

1.3.1 Fermi-Dirac distribution

Fermi-Dirac distribution function: Is a statistical function which gives the probability that an available energy states will be occupied by an electron at an absolute temperature T (k^o)

$$f(E) = 1 / (1+e^{(E - Ef)/ kt})$$

Where k = Boltzmann constant

E_f = Fermi level (very importance reference point)

Observations:

$$f(E) = \frac{1}{2}$$

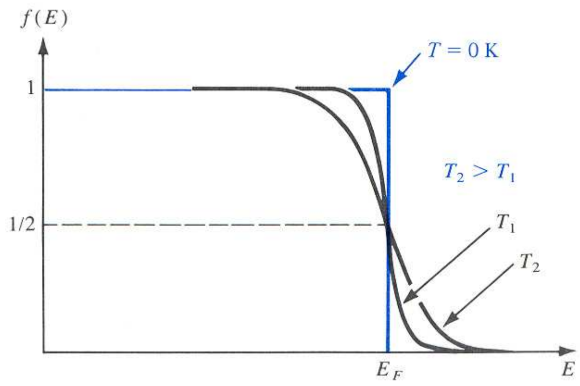
$E = 0 \Rightarrow f(0) \approx 1$, $E = \infty \Rightarrow f(\infty) = 0$

$T= 0 \Rightarrow f(E<E_f) =1/(1+e^{-\alpha}) =1$, $E<E_f$

$T=0 \Rightarrow f(E>E_f) = 1/(1+e^{\alpha})$, $E>E_f$

$T=o \Rightarrow f(E) = 1/(1+e^0) = \frac{1}{2}$

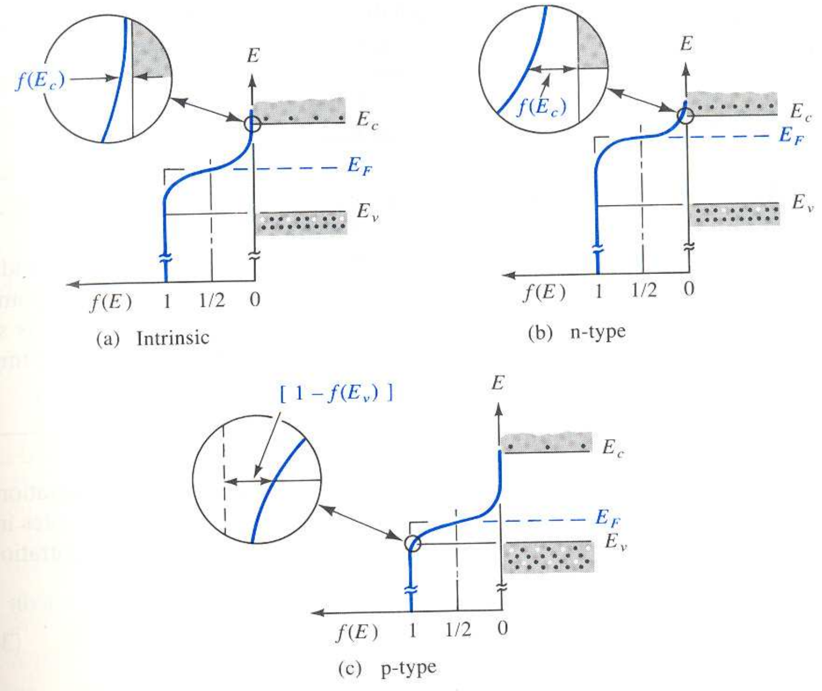
Observation: Plot of f(E) as the probability of occupancy of an available state at E is in the band gap. There is no available state and no electrons.



Mathematically symmetry can be shown:

$$f(E_f + \Delta E) = 1 - f(E_f - \Delta E)$$

Position of E_f for semiconductor:



Common practice: Is not to draw f(E) vs. E on every energy band diagram but merely to indicate the position of E_f in the band diagram.

1.3.2 Electron and hole concentration at equilibrium

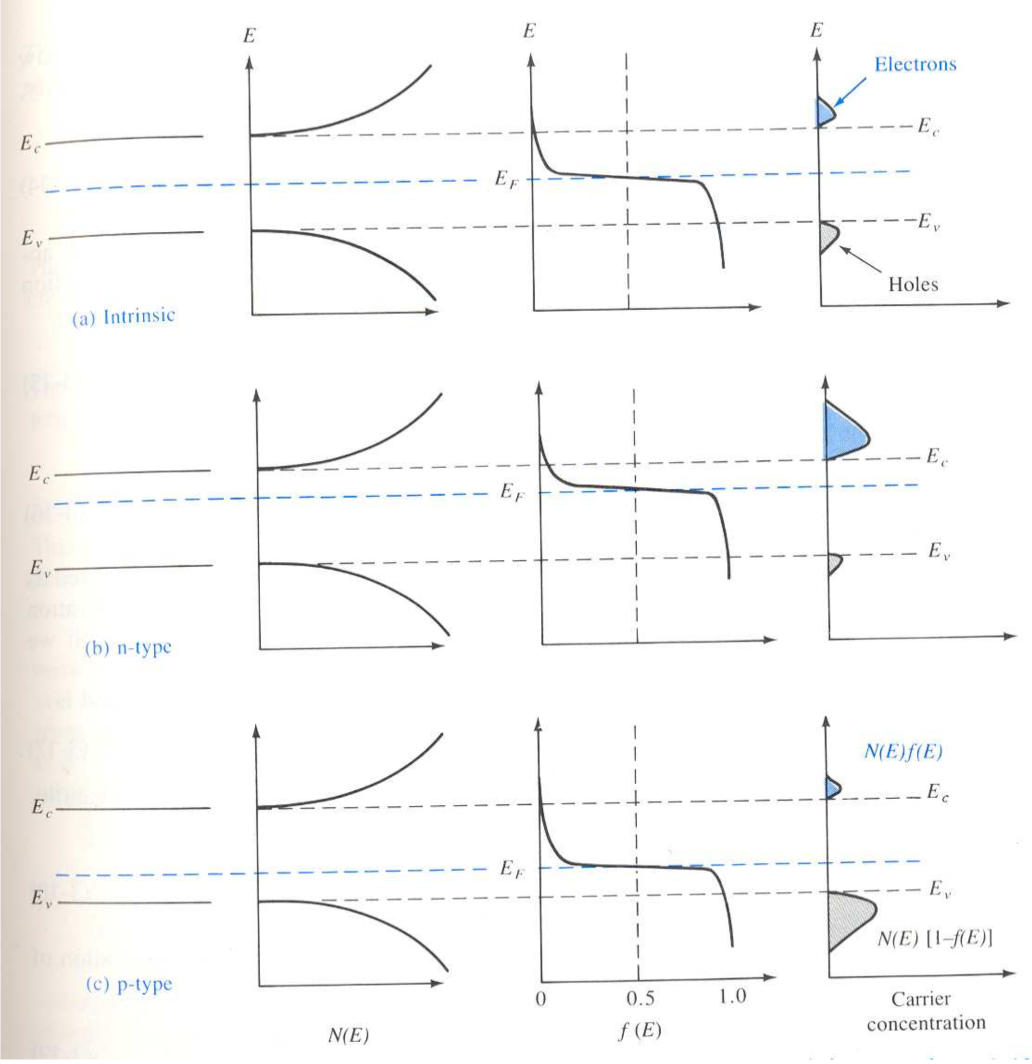
Using f(E) , n_o and p_o can be found

$$n_o = \int_{Ec}^{\infty} f(E)N(E)dE \qquad \text{--------------------------equation 1.3}$$

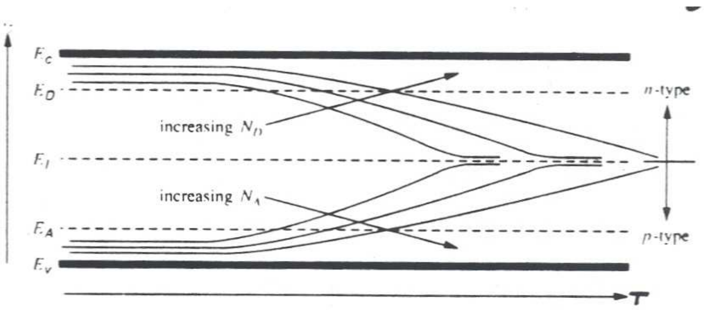
$$p_o = \int_{0}^{Ev} [1 - f(E)]N(E)dE \qquad \text{-----------------------equation 1.4}$$

where n(E)dE is the density of states in the energy range dE.

Fact: It is shown mathematically that N(E) is $\alpha\sqrt{E}$ on one hand f(E) decreases very rapidly for higher energies. As a result f(E)N(E) decreases above E_c for n_o and [1- f(E)]N(E) decreases below E_v for p_o



FACT: E_F varied with Temp. for Extrinsic s/c as shown →



Fermi energy as a function of temperature (arbitrary units) for various impurity concentrations. The temperature dependence of the band gap is neglected.

Simplification:

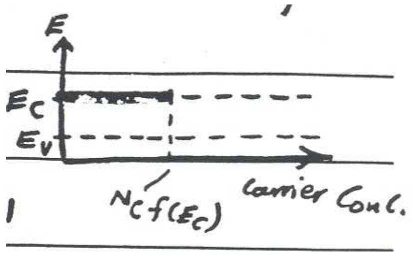
Electron Concentration: If we represent all of the distributed electrons states in the conduction band by an effective density of state (N_c) located at E**c.**

then $n_o = N_c f(E_c)$

where $f(E_c) = e^{-(Ec-Ef)/kt}$ for $(E_c-E_f)/kt >>1$

$N_c = 2(2\prod m_n^{*} kt/h^2)^{3/2}$

Thus: $\mathbf{n_o = N_c e^{-(Ec-Ef)/kt}}$ ----------------------------------- equation 1.5



Similarly for hole concentration :

$P_o = N_v [1-f(E_v)]$

Where $1-f(E_v) = e^{-(Ef-Ev)/kt}$ for $|(E_v - E_f)/kt|$

$N_v = 2(2\prod m_p^{*} kt/h^2)^{3/2}$

Thus: $\mathbf{P_o = N_v e^{-(Ef-Ev)/kt}}$ ---------------------------------------equation 1.6

Observation: The two equations above (1.5 and 1.6) are valid for intrinsic or doped materials at thermal equilibrium. They are general equations for carrier concentrations.

1.3.3 Further Semiconductor Mathematics

For intrinsic semiconductor : $E_f = E_i$ (middle of the band gap approximately since density of states in conduction band and valence band are not equal E_f is not exactly at the center of band gap)

Intrinsic: $n_i = N_c e^{-(Ec - Ei)/kt}$

$P_i = N_v e^{-(Ef-Ei)/kt}$

Then we get $n_i p_i = N_c N_v e^{-Eg/kt}$

Note: $E_c - E_v = E_g$

Doped:

$n_o = N_c\, e^{-(Ec-Ef)/kt}$

$P_o = N_v\, e^{-(Ef-Ev)/kt}$

Then we get $n_o\, p_o = N_c\, N_v\, e^{-Eg/kt}$

Since $n_i = P_i$ ➔ $n_i = \sqrt{NcNv}\ e^{-Eg/2kt}$ ------------------------- equation 1.7

And $\mathbf{n_o p_0 = n_i^2}$

And $n_i = N_c\, e^{-(Ec - Ei)/kt}$

$P_i = N_v\, e^{-(Ef-Ei)/kt}$

We get $\frac{no}{ni} = e^{(Ef - Ei)/kt}$

Similarly $\frac{Po}{Pi} = e^{-(Ef - Ei)/kt}$

Ex 1: Consider a si sample with $N_d = 10^{17}$ As atoms/cm^3. Find P_o at $T = 300^o$ K? What is $E_f - E_i$?

Solution: $n_i = 1.5 \times 10^{10}$, since $N_d >> n_i$ therefore $n_0 = N_d = 10^{17}$ / cm^3

$P_o = n_i^2 \div n_0 = 2.25 \times 10^3$ cm^{-3}

We know that $E_f - E_i = kt \ln(n_o/n_i) = 0.0259 \ln(10^{17}/1.5 \times 10^{10}) = 0.407$ ev

1.3.4 Dependence of "n" on temperature

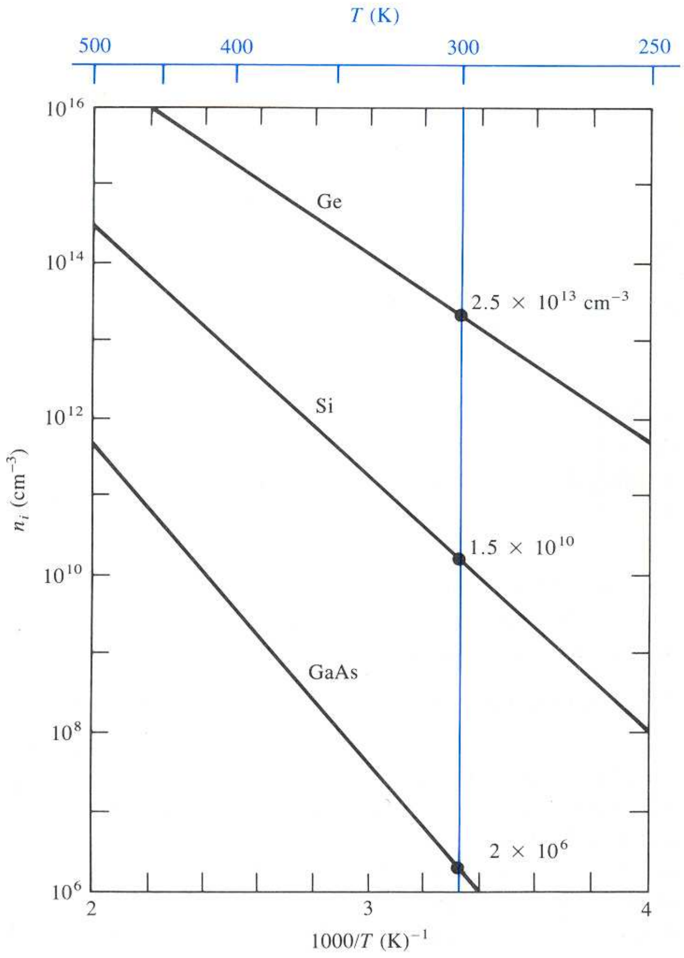
Intrinsic:

We know from equation 1.7 $n_i = \sqrt{NcNv}\ e^{-Eg/2kt}$

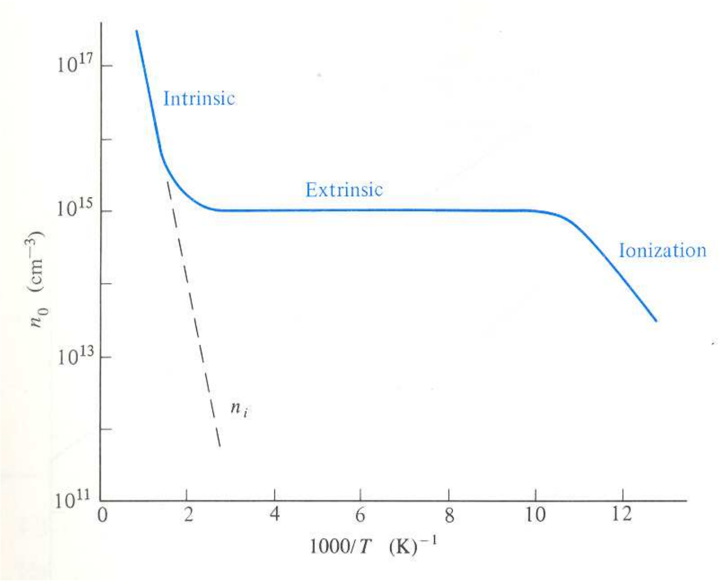
Substituting for N_c and N_v

$n_i(T)^2 = (2\prod kt/h^2)^{3/2}\,(m_n^* m_p^*)^{3/4}\, e^{-Eg/2kt}$ --------------- equation 1.10

The exponential term dominates the temperature dependence equation shown be



Extrinsic:



Observation: for the temperature range of $2.0 \le 1000/T \le 10$ ➔ $100 < T \le 500$ K

The material carrier concentration is controlled by doping level primarily. Outside this range EHP generation is done thermally and is not desirable!

1.3.5 Principle of charge compensation and space charge neutrality compensation

When both donors and acceptors are present the predominance of one of the dopants over the other one cause the material to the either p or n type as follows:

$N_d >> N_a$ ➔ n-type (donor electrons compensated and exceed acceptor holes)

$N_a >> N_d$ ➔ p-type (acceptor holes compensated and exceed donor electrons)

Space charge neutrality:

$P_o + N_d^+ = n_o + N_a^-$ -----------eq 1.11 , i.e. all positive charges = all negative charges

To find n_o and p_o , recall $n_o p_o = n_i^2$

Eliminate p_o in eq1.11 ➔ $n_o = n_i^2/n_o + (N_d - N_a)$

Assume $N_d > N_d$ ➔ $n_o^2 - n_o N_{eff=} - n_i^2 = 0$, $N_{eff} = N_d - N_a > 0$

$\Rightarrow$, $n_o = \frac{1}{2}[N_{eff} + (N_{eff}^2 + 4n_i^2)^{1/2}]$ ------------eq 1.12 (n-type semiconductor)

And $p_o = n_i^2/n_o$

Approximation:

If $N_{eff} >> n_i \Rightarrow n_o \approx N_d - N_a \approx N_d$ ---------------eq 1.1.2a (if $N_a = 0$,it is n-type)

Note: If $N_a > N_d$, find p_o first.

$P_o = n_i^2/p_o + (N_a - N_d) \Rightarrow p_o^2 - p_oN_{eff}' - n_i^2=0$

$P_o = \frac{1}{2}[N_{eff}^I + (N^I_{eff}{}^2+4n_i^2)^{1/2}]$, $N^I_{eff} = N_a - N_d >0$

And $n_o = n_i^2/p_o$, $N^I_{eff} = -N_{eff}$

Approximation:

If $N^I{}_{eff} >> (p_i = n_i) \Rightarrow p_o \approx N_a - N_d \approx N_a$ (if $N_d \approx 0$)

$N_o = n_i^2/N_a$ p-type

1.4- Drift of Carries in an E-field

Observation: Knowledge of carrier concentrations in a solid is necessary for calculating current flow in an E-field. In addition to the values of n and p, the ease of motion, velocity of the carriers and their collisions and scatterings with the lattice and impurities must be taken into account. The last consideration brings about the concepts of mobility and conductivity as described next.

1.4.1-Mobility and conductivity:

The individual electrons and holes move in many directions by thermal motion during a given period of time. On average they have a constant net drift velocity (which is must smaller than the random speed due to thermal motion).

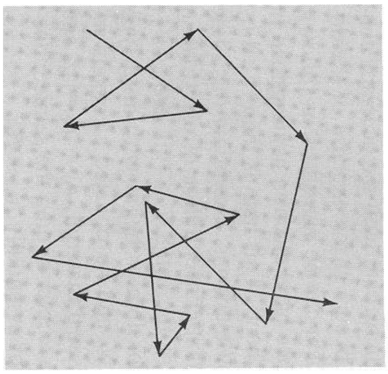
$\bar{v}_n = -\mu_n\bar{\varepsilon}$ and $\bar{v}_p = \mu_p\bar{\varepsilon}$ ------------------------- eq 1.14

Where $\mu_n = e^-$ mobility $= q\bar{t}/m^*_n$ and $\mu_p = h^+$ mobility $= q\bar{t}/m^*_p$

$\bar{t}$ = mean free time (mean time between collision events)

Definition:

1. **Mobility-** The average particle drift velocity per unit electric field.



Drift of an electron and their motion.

1.4.2- Current Flow:

Current density (defined to be current per unit area) is the number of carriers crossing a unit area per unit time ($n\bar{v}_n + p\bar{v}_p$) multiplied by the charge on the carriers

$$\bar{J} = qn\mu_n\bar{\varepsilon} + qp\mu_p\bar{\varepsilon} = q\,(n\mu_n + p\mu_p)\,\bar{\varepsilon} = \sigma\bar{\varepsilon}$$

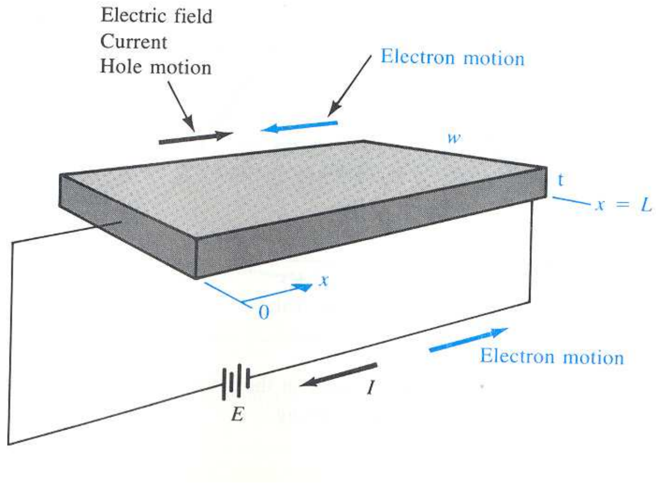
Where $\sigma = q\,(n\mu_n + p\mu_p)$ is called conductivity

$\bar{J} = \sigma\bar{\varepsilon}$ ohm's law in point form. ------------------eq 1.15

$I\bar{J}I = \frac{I}{A}$ and $v = I\varepsilon I\ L$

From eq 1.15 $\frac{I}{A} = \sigma\frac{V}{L} \Rightarrow v = \frac{L}{\sigma A}\, I = RI$ ----------------- eq 1.16 (ohm's law in integral form)

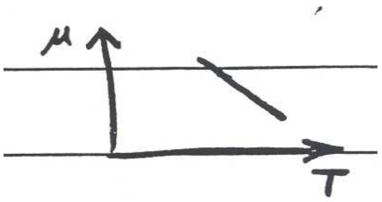
Where $R = \frac{L}{\sigma A} = \frac{\rho L}{A}$, $\rho = \sigma^{-1}$ = resistivity



1.4.3- Effects of Temperature and Doping on Mobility:

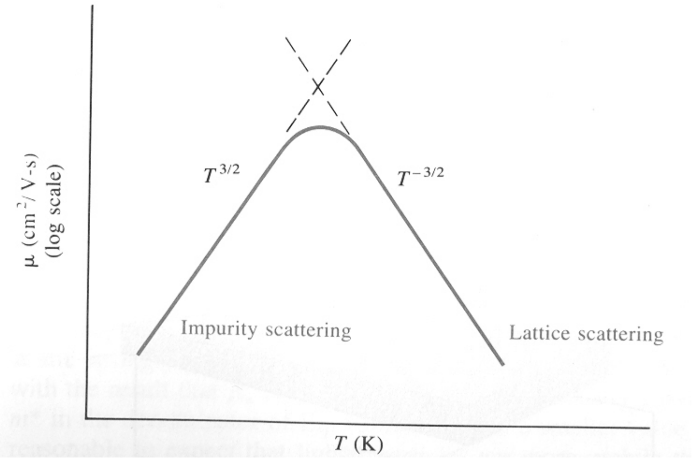
Fact: There are two basic types of scattering mechanism that influence μ_n and μ_p

1. **Lattice Scattering:** (phonon scattering) Scattering of a carrier (while moving through the crystal) due to vibrations of the lattice, resulting from the temperature. Mobility decreases as the sample is heated.

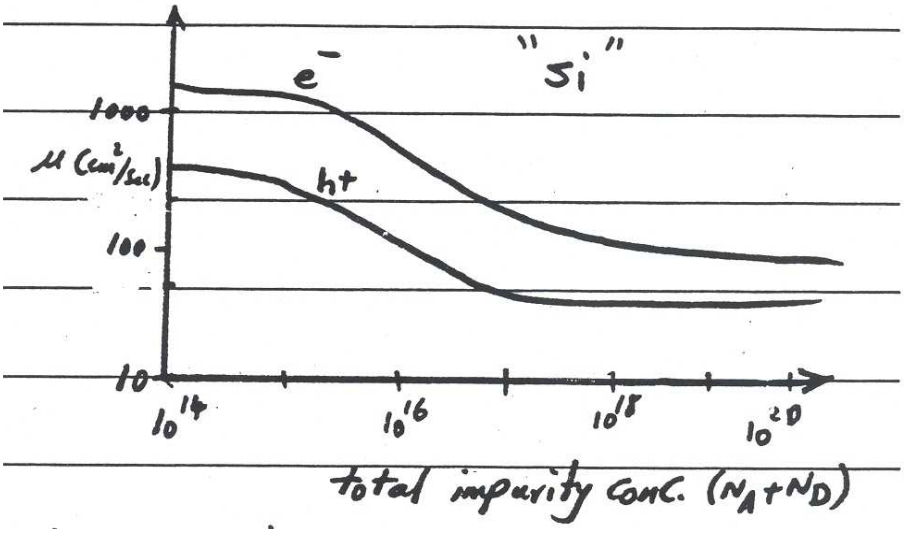


2. **Impurity Scattering:** Scattering from crystal defects such as ionized impurities. This type would be dominated at low temperature, since a slowly moving carrier is likely to be scattered more strongly by an interaction of a charged ion then a carrier with greater velocity. Since the atoms of the lattice are less agitated at low temperature, lattice scattering thermal motion of carriers is also slower.

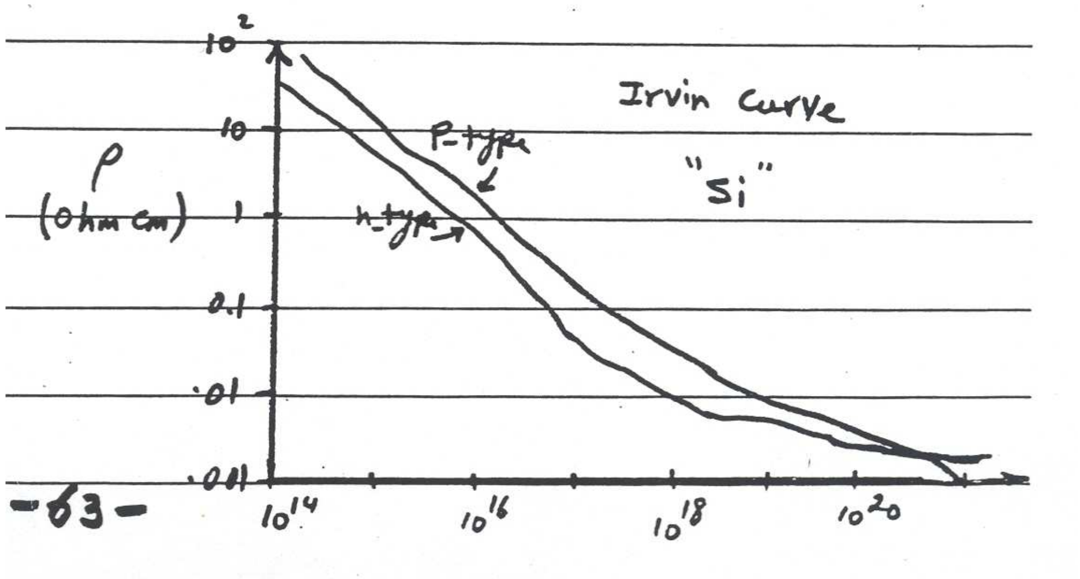
$$\frac{1}{\mu total} = \frac{1}{\mu i} + \frac{1}{\mu l} \quad \text{-------------------------} \quad \text{eq 1.17}$$



As the concentration of impurities increases, the effects of impurity scattering are felt at higher temperatures. Not only is mobility different for each type of carrier (n or p), but also as the impurity concentration increases. This decrease is a function only of the total amount of impurity and not of their type of charge.

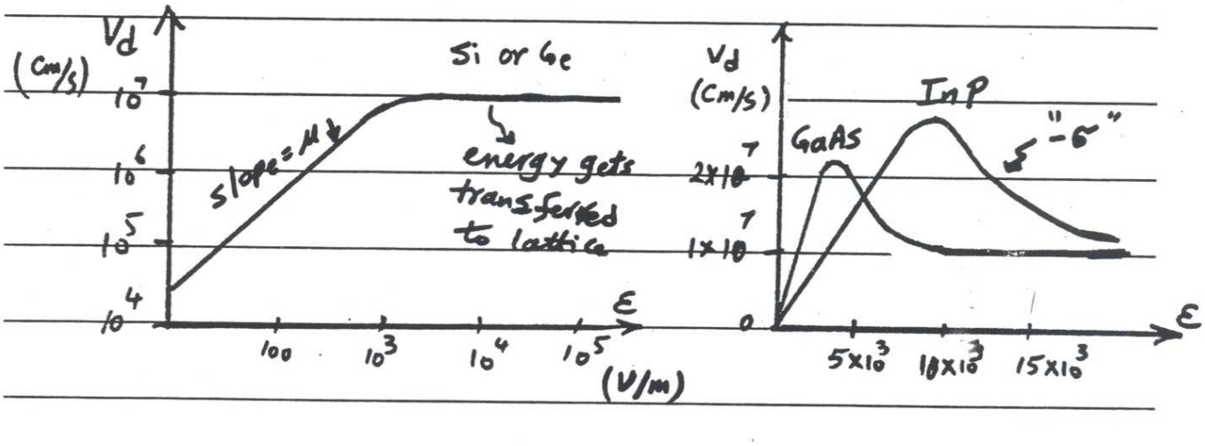


A resistivity curve done by Irvin has become a standard reference work in semiconductor technology.



1.4.4- High E-field Effects

Ohm's law (equation 1.15) is valid only for lower E-field (<10^3 v/cm) in semiconductor. Above this approximate value "σ" becomes a function of E-field and is not a constant any more. This is referred to as the “hot carrier effect.” In this region drift velocity saturates to a value comparable to the thermal velocity (10^7 cm/s). This behavior is typical of Si and Ge, etc. Some semiconductors like GaAs exhibit negative conductivity at high fields.



1.5-Invariance of the Fermi level at equilibrium:

In the following chapters we will be considering cases in which non-uniform doping in a semiconductor or junctions occur between different semiconductors or a semiconductor and a metal. These cases are crucial to the various types of electronic opto electronic devices made in semiconductors.

Analysis: For two materials in intimate contact (such that electron can move between the two) there is no current and therefore no net charge or energy transport at thermal equilibrium. Since there is no net energy transfer any transfer of electron from material 1, to material 2 must be exactly balanced by the opposite transfer of electron from 2 to 1.

Calculations lead to $f_1(E) = f_2(E)$ -- 1.18a

There is no discontinuity in the equal Fermi level ,which means

$$\frac{dEf}{dx} = 0 \quad \text{(no gradient!)} \text{ ------------------------------ } 1.18b$$

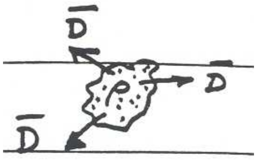
Conclusion:

Rule: No discontinuity or gradient can arise in the equilibrium Fermi level E_f .

Material 1	Material 2
Semiconductor 1	Semiconductor 2
Semiconductor	Metal
n-type semi conductor	p-type semi conductor
$f_1(E)$	$f_2(E)$
E_{F1}	E_{F2}

1.6- Fundamental Equations For Semiconductors

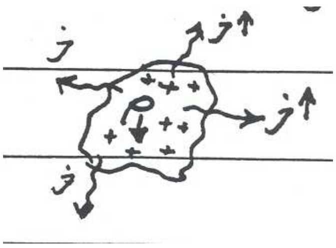
1. $\nabla.\overline{D}$ = ρ(x,y,z) (electric Gauss' law) ------------------- 1.19



$\overline{D}$ = ε$\overline{E}$ = electric displacement vector

$\overline{E}$ = electric field vector

2. $\nabla.\overline{J}_{cond} = -\frac{\partial}{\partial t}\rho(x,y,z)$ (law of conservation of charge)



$\overline{J}_{cond}$ = It is the conduction current density

3. Use equation 1.19 to obtain:

$$\nabla.\bar{\varepsilon} = \frac{\rho}{E}$$

$$\bar{\varepsilon} = -\nabla V$$

$$\nabla^2 V = -\frac{\rho}{E}$$ (poisson’s equation) ----------------- 1.21

$$\nabla^2 = \frac{\partial 2}{\partial x2} + \frac{\partial 2}{\partial y2} + \frac{\partial 2}{\partial z2}$$

4. Current Density Equations:

$J_n = q\mu_n n\varepsilon + qD_n \nabla n$
$j_p = q\mu_p p\varepsilon - qD_p \nabla p$
$j_{cond} = j_n + j_p$ --------------1.22

5) $\partial n/\partial t = g_n - r_n + \frac{1}{q} \nabla .j_n$

$$\frac{\partial p}{\partial t} = g_p - r_p - \frac{1}{q} \nabla .j_p$$ continuity equation -------- 1.23

Note: 1. $\nabla = \frac{\partial}{\partial x}\hat{X} + \frac{\partial}{\partial y}\hat{Y} + \frac{\partial}{\partial z}\hat{Z}$ is the Del operator

Equation $\nabla .\hat{D} = \frac{\partial Dx}{\partial x} + \frac{\partial Dy}{\partial y} + \frac{\partial Dz}{\partial z}$ (divergence) ; ∇V (gradient)

$\nabla.\nabla = \nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ (laplacian) = Divergence of gradient.

1.7 SELECTED EXAMPLES

The following examples are selected from several texts; therefore there may be some notation differences which should be kept in mind.

CHAPTER 1 EXAMPLES

EX1) Chapter 1

A silicon sample is doped with 10^{17} As/cm^3.

Find equilibrium hole concentration (p_o) ?

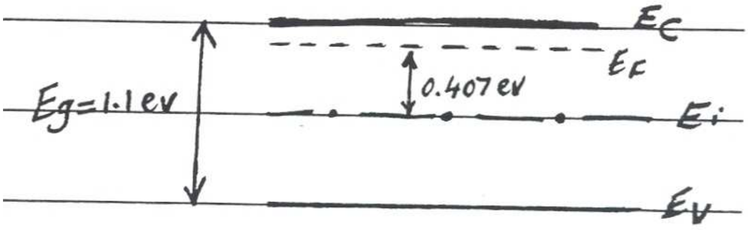
Where is E_f relative to E_i ?

Solution:

since $N_d >> n_i = 1.5\times 10^{10} \Rightarrow n_o = N_d = 10^{17}/cm^3$

$p_o = n_i^2/n_o = 2.25\times 10^3$

$E_f - E_i = KT \ln (n_o/n_i) = 0.407$ ev



EX2) Find the resistivity of the intrinsic Ge at 300^{o}k?

Solution: intrinsic material $\Rightarrow n_o = p_o = n_i = 2.5\times 10^{13}/cm^3$

From appendix $\mu_n = 3900$, and $\mu_p = 1900$ cm^2/v-s

$\sigma_i = q(\mu_n + \mu_p)n_i = 2.32\times 10^{-2}\ (\Omega\ cm)^{-1}$

$\rho_i = 1/\sigma_i = 43$ Ώ-cm

EX 3) Chapter 1: (given below)

______________ Find the proper directions, signs for the currents, drift velocities, and mobilities produced by the motion of both negatively and positively charged particles moving in the same medium under the application of a constant external electric field $\mathscr{E}$.

Solution. Consider a region of a medium where due to the application of a potential difference v, an electric field $\mathscr{E}$ is produced, as shown in Fig. E.4.1. The conventional electrical current flows from the positive toward the negative terminal.

From Fig. E.4.1, $\mathscr{E} = \mathscr{E}_0(-\mathbf{a}_x)$, where $\mathscr{E}_0 = v/L$ and the corresponding velocities are given by

$$\begin{aligned} \mathbf{v}_- &= v_-\mathbf{a}_x \qquad \text{(electron)} \\ \mathbf{v}_+ &= v_+(-\mathbf{a}_x) \qquad \text{(holes)} \end{aligned} \tag{E.4.1}$$

where v_- and v_+ are the magnitudes of the respective velocity vectors. By definition, the current density is given by (Eq. 1.15a) Using $q = -e$ for an electron and $q = +e$ for a singly charged positive charge, respective current densities are

$$\mathbf{J}_- = -en_-\mathbf{v}_- = -en_-(v_-\mathbf{a}_x) \tag{E.4.2a}$$

$$= en_-v_-(-\mathbf{a}_x) \qquad \text{(electrons)}$$

$$\mathbf{J}_+ = en_+\mathbf{v}_+ = en_+(-v_+\mathbf{a}_x) \tag{E.4.2b}$$

$$= en_+v_+(-\mathbf{a}_x) \qquad \text{(positive charge)}$$

We see that both current densities are in the same direction, and the total current density is

$$\begin{aligned} \mathbf{J} &= \mathbf{J}_- + \mathbf{J}_+ \\ \mathbf{J} &= (en_-v_- + en_+v_+)(-\mathbf{a}_x) \end{aligned} \tag{E.4.3}$$

$\mathbf{J}$, given by Eq. E.4.3, has the right sign of the conventional electrical current. The drift velocity is related to the electric field by

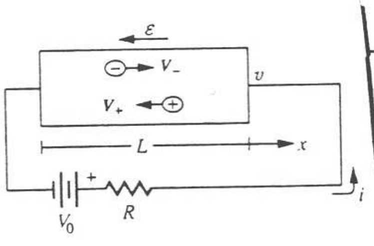
$$\mathbf{v} = \mu\,\mathscr{E} \tag{E.4.4}$$

From Fig. E.4.1, $\mathscr{E} = \mathscr{E}_0(-\mathbf{a}_x)$ and from the corresponding velocities, the sign of mobilities is

$$v_-\mathbf{a}_x = \mu_-E_0(-\mathbf{a}_x) \qquad \mu_- = -\mu_\text{n} \qquad \text{(electron)} \tag{E.4.5a}$$

$$v_+(-\mathbf{a}_x) = \mu_+E_0(-\mathbf{a}_x) \qquad \mu_+ = +\mu_\text{p} \qquad \text{(positive charge)} \tag{E.4.5b}$$

Fig. E4.1

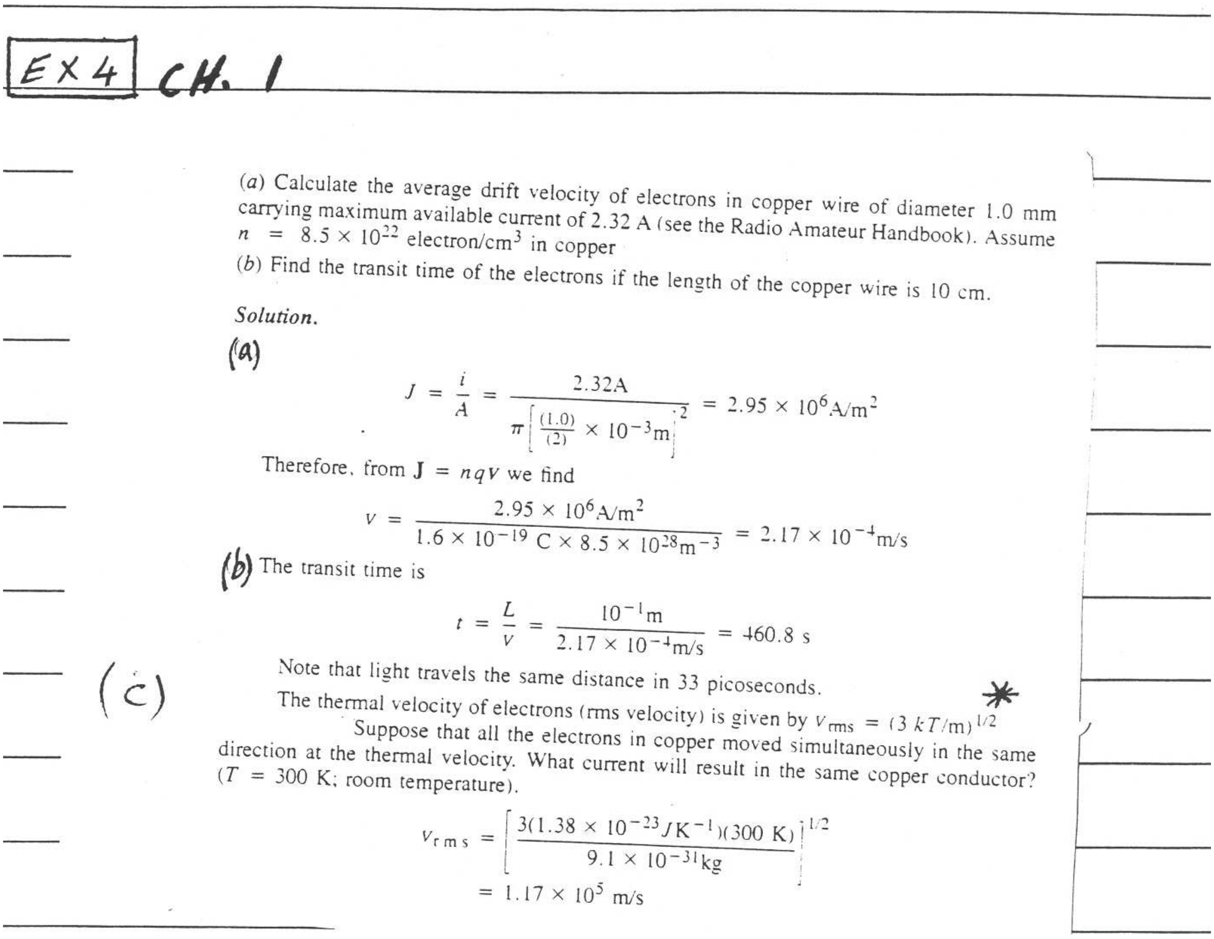


Motion of negative and positive charges under the application of an $\mathscr{E}$ field.

Similarly, the conductivity due to both charges is

$$\begin{aligned} \sigma &= q_-n_-\mu_- + q_+n_+\mu_+ \\ &= -en_n(-\mu_n) + en_p(+\mu_p) \\ &= en_n\mu_n + en_p\mu_p \end{aligned} \tag{E.4.6}$$

Here n_n and n_p stand for the negative- and positive-particle densities. The current density and the conductivity of the sample arise from the contributions of the two charged species, although the particles move in opposite directions.



EX 4 CH. 1

(a) Calculate the average drift velocity of electrons in copper wire of diameter 1.0 mm carrying maximum available current of 2.32 A (see the Radio Amateur Handbook). Assume $n = 8.5 \times 10^{22}$ electron/cm³ in copper

(b) Find the transit time of the electrons if the length of the copper wire is 10 cm.

Solution.

(a)

$$J = \frac{i}{A} = \frac{2.32\text{A}}{\pi\left[\frac{(1.0)}{(2)} \times 10^{-3}\text{m}\right]^2} = 2.95 \times 10^6 \text{A/m}^2$$

Therefore, from $\mathbf{J} = nqv$ we find

$$v = \frac{2.95 \times 10^6 \text{A/m}^2}{1.6 \times 10^{-19}\text{ C} \times 8.5 \times 10^{28}\text{m}^{-3}} = 2.17 \times 10^{-4}\text{m/s}$$

(b) The transit time is

$$t = \frac{L}{v} = \frac{10^{-1}\text{m}}{2.17 \times 10^{-4}\text{m/s}} = 460.8 \text{ s}$$

Note that light travels the same distance in 33 picoseconds.

(c) The thermal velocity of electrons (rms velocity) is given by $v_{rms} = (3\,kT/m)^{1/2}$ * Suppose that all the electrons in copper moved simultaneously in the same direction at the thermal velocity. What current will result in the same copper conductor? ($T = 300$ K; room temperature).

$$v_{rms} = \left[\frac{3(1.38 \times 10^{-23} J\,K^{-1})(300\text{ K})}{9.1 \times 10^{-31}\text{kg}}\right]^{1/2}$$

$$= 1.17 \times 10^5 \text{ m/s}$$

Note: $E = \frac{1}{2} mV^2$

$E = \frac{3}{2} KT$

Therefore $V = (2E/m)^{1/2} = (3kt/m)^{1/2}$

The resulting current density will be:

$J_{th} = qnv_{rms} = 1.59 \times 10^{15}$ A/m²

$I = j_{th} A = 1.25 \times 10^9$ A quite high!

Current is high due to our assumption of all electrons move simultaneously at v_{rms}.

EX 5 CH.1

Calculate the intrinsic hole concentration of a germanium crystal at room temperature.

Solution. For simplicity, we take $m_e^* = m_p^* = m_e$ (the free electron mass). The band gap energy of germanium is 0.67 eV.

eq. (1.10) $\Rightarrow$

$$p = n_i = \left(\frac{2\pi\ 1.38 \times 10^{-23}\ \text{J/K}}{(6.62 \times 10^{-34}\ \text{J}\cdot\text{s})^2}\right)^{3/2} [(9.1 \times 10^{-31}\ \text{kg})^2]^{3/4}$$

$$\times\ (300\ \text{K})^{3/2} \exp\left[-\frac{(0.67\text{eV})(1.6 \times 10^{-19}\ \text{J/eV})}{2(1.38 \times 10^{-23}\ \text{J/K})(300\ \text{K})}\right]$$

$$p = 5.98 \times 10^{22}\ \text{holes/m}^3 = 5.98 \times 10^{16}\ \text{holes/cm}^3$$

Note :

In an intrinsic semiconductor

$$n = p = n_i$$

Thus, the intrinsic electron or hole concentrations are given by

$$n_i = 2\left(\frac{2\pi k}{h^2}\right)^{3/2} (m_e^* m_p^*)^{3/2} T^{3/2} e^{(-E_g/2kT)}$$

EX 6 CH. 1

Calculate the electron density in an n-type silicon semiconductor, if the donor-atom density is $5 \times 10^{16}\ \text{cm}^{-3}$. Assume that 90 percent of the donor atoms are ionized. Also find the hole density in this n-type semiconductor.

Solution. For silicon $n_i = 1.4 \times 10^{10}\ \text{cm}^{-3}$. We can therefore neglect n_i in (Eq. 1.12a)

$$n = n_D^+ = 0.9N_D = 4.5 \times 10^{16}\ \text{cm}^{-3}$$

From Eq.6.8, the hole density is

$$p = \frac{n_i^2}{n} = 4.36 \times 10^3\ \text{holes/cm}^3$$

Note that the hole density in an n-type semiconductor falls way below the intrinsic hole density.

EX 7 CH. 1

For Example 6, find the location of (a) the Fermi level E_F and (b) the donor level E_D.

Solution. Using Eq. 1.9, we can write

$$N_c \exp\left(\frac{E_F - E_c}{kT}\right) = n_D^+ = 0.9N_D$$

Solving for E_F, and using $N_c = 2.5 \times 10^{19}$ cm^{-3}, we find

$$E_F - E_c = kT \ln\left(\frac{0.9N_D}{N_c}\right)$$

$$E_F - E_c = (0.026) \ln\left[\frac{(0.9)5 \times 10^{16}}{2.5 \times 10^{19}}\right] \text{ eV}$$

Finally

$$E_F = E_c - 0.164 \text{ eV}$$

The Fermi level is 0.164 eV below the conduction band.

we know : $N_D^+ = n_D^+ = N_D[1 - f(E_D)]$ = ionized Donor Conc. *

$(N_D = N_d)$

$$n_D^+ = N_D\left\{\frac{1}{1 + \exp\left[(E_F - E_D)/kT\right]}\right\}$$

Setting

$$n_D^+ = 0.9N_D$$

and solving for E_D and using $E_F = E_c - 0.164$ eV from the above solution, we find

$$E_D = E_c - 0.107 \text{ eV}$$

The donor energy is 0.107 eV below the conduction band.

Note : $N_a^- = n_a = N_a f(E_A)$ = ionized acceptor concentration

In general equation 1.11 for charge neutrality becomes

$$n + N_A = p + N_d$$

$$\Rightarrow N_C e^{(E_F - E_C)/kT} + N_a\left[\frac{1}{1 + e^{(E_A - E_F)/kT}}\right] = N_V e^{-\frac{(E_F - E_V)}{kT}} + N_d\left[\frac{1}{1 + e^{(E_F - E_D)/kT}}\right]$$

EX8 CH.1

Determine the probability that an energy level is occupied by an electron at 300°K if it is located above the Fermi level by

(a) 0.026 eV ($=kT$),
(b) 0.078 eV ($=3kT$),
(c) repeat parts (a) and (b) at 600°K.

Solution

From Eq. (1.2)

(a)

$$f(E)|_{0.026\ \text{eV}} = \frac{1}{e^{kT/kT}+1} = \underline{0.27},$$

(b)

$$f(E)|_{0.078\ \text{eV}} = \frac{1}{e^{3kT/kT}+1} = \underline{0.05}.$$

The latter result indicates that at room temperature, any energy level only 0.078 eV above the Fermi energy level has a probability of 1 in 20 of being occupied. At the Fermi energy this probability of occupancy is 1 in 2.

(c) At 600°K, 0.026 eV = $\tfrac{1}{2}kT$ and 0.078 eV = $\tfrac{3}{2}kT$. Hence at 600°K

$$f(E)|_{0.026\ \text{eV}} = \frac{1}{e^{(1/2)kT/kT}+1} = \underline{0.38}.$$

$$f(E)|_{0.078\ \text{eV}} = \frac{1}{e^{(3/2)kT/kT}+1} = \underline{0.18}.$$

Hence the occupation probability increases substantially with temperature increase.

EX 9 CH.1

Intrinsic semiconductor material A has an energy gap of 0.36 eV, while material B has an energy gap of 0.72 eV. Compare the intrinsic density of carriers in these two semiconductor materials at 300°K. Assume that the effective masses of all the electrons and holes are equal to the free electron mass.

Solution

Using Eq. (1.10) we have

$$\frac{n_{iA}}{n_{iB}} = \frac{e^{-E_{gA}/2kT}}{e^{-E_{gB}/2kT}} = e^{(E_{gB}-E_{gA})/2kT}$$

$$= e^{(0.72-0.36)\ \text{eV}/0.052\ \text{eV}}$$

$$= \underline{1000}.$$

Hence, although the energy gap of these two intrinsic semiconductors differs only by a factor of 2, the intrinsic density of carriers in the narrower gap material is 1000 times greater than that in the wider gap semiconductor.

EX 10

CH. 1

Given the n-type semiconductor silicon at 300°K with an energy gap of 1.12 eV. The material contains only donor-type impurities, all of which are ionized. The donor density is $1.0 \times 10^{16}/\text{cm}^3$. Calculate the Fermi energy.

Solution

Beginning with the charge neutrality condition given in Eq. (4.34), since no acceptors are present and all donors are ionized, $n_d = 0$ and $N_a - p_a = 0$. This equation then reduces to

$$p + N_d = n.$$

Since the np product for silicon is $(1.5 \times 10^{10})^2/\text{cm}^6$ and there must be at least $10^{16}/\text{cm}^3$ electrons in conduction, the hole concentration must be less than $[(1.5 \times 10^{10})^2/10^{16}]/\text{cm}^3$. Hence $p \ll n, N_d$. So to a good approximation

$$N_d = n,$$

or essentially all the conduction electrons come from ionized donors. Using Eq. (4.21a) we have

$$N_d = M_c \times 2(2\pi m_n^* kT/h^2)^{3/2} e^{-(E_c - E_F)/kT}$$

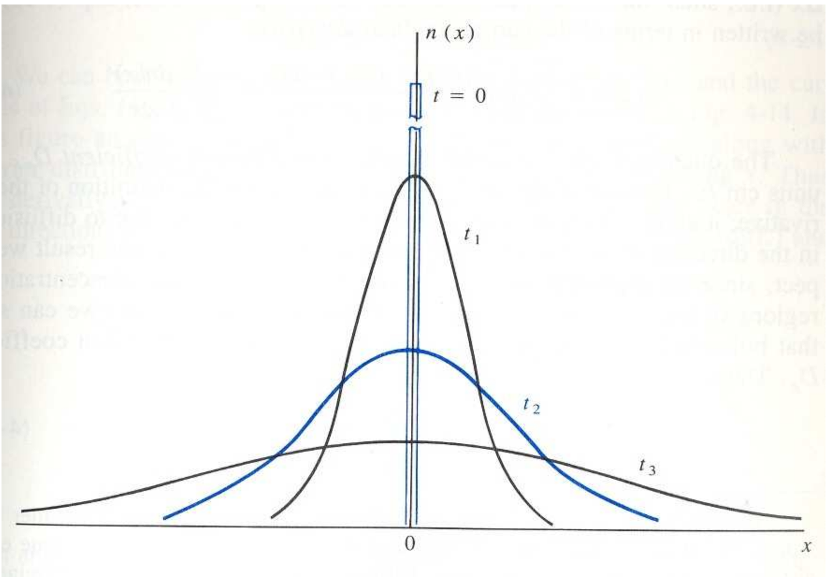
$$= 6(4.82 \times 10^{15})(m_n^*/m_0)^{3/2} T^{3/2} e^{-(E_c - E_F)/kT}. = 10^{16}$$

$$\Longrightarrow E_C - E_F = 0.21\ \text{eV}$$

CHAPTER 2 – Excess Carriers in Semiconductor

2.1- Diffusion: When excessive carriers create non-uniformly in a semiconductor, the electrons and holes concentrations vary with position in the sample. Any such spatial variation (gradient) in “n” and “p” calls for a net motion of the carriers from region of high concentration to region of low concentration. This type of motion is called **diffusion** and represents an important charge transport process in semiconductor.

Observation: The diffusion is the natural result of the random motion of the individual charge carriers due to the thermal energy present.



Ex: Spreading of a pulse of electrons by diffusion.

Observation: The current flow is proportional to the gradient decrease (or slope) of the concentration, thus

$(j_n)_{diff} = q\,D_n\,dn/dx$

$(J_p)_{diff} = -q\,D_p\,dp/dx$

Note: Electrons and holes diffusing in the same direction, causes opposite currents.

2.2 - Drift and diffusion of carriers:

The two basic processes of current conduction are

1. Diffusion due to a carrier gradient.
2. Drift due to an electric field.

Therefore: $j_n(x) = q\mu_n n(x)\varepsilon(x) + qD_n dn(x)/d(x)$ ---------------- 2.2a

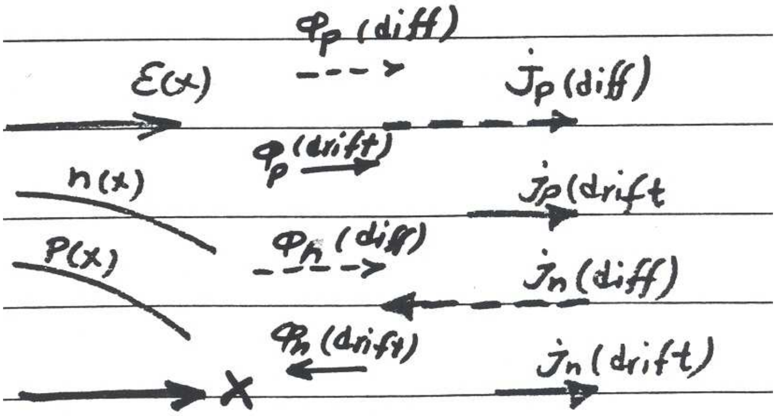
Drift   Diffusion

$$J_p(x) = q\mu_p p(x)\varepsilon(x) - qD_p dp(x)/d(x) \qquad \text{-----------------}\ 2.2b$$

The total current density is

$$J(x) = j_n(x) + j_p(x)$$

$$J(x) = q(\mu_n n(x) + \mu_p p(x))\varepsilon(x) + q(D_n dn(x)/d(x) - D_p dp(x)/d(x))$$



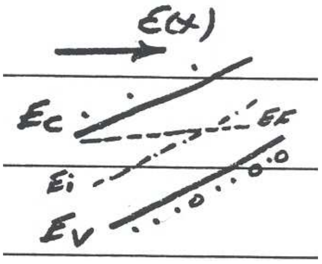
Observation:
Minority carriers can contribute significantly to the current through diffusion (for a significant gradient, dk/ dx or dp/dx), since drift term is proportional to n or p and is thus small.

2.2.1- Influence of E-field on energy bands:

Observation: Since the electron drift in a direction opposite to the field, the band energy for electrons should increase in the direction of the field. However the electrostatic potential function varies in the opposite direction because:

$$, \varepsilon(x) = - dv(x)/d(x) = - d/dx(E_i/-q)$$

$$, \varepsilon(x) = 1/q.dE_i/dx$$



Note: E_i is chosen as a convenient reference to calculate V(x) since we are interested only in spatial variation of V(x).

Point of interest:

To remember the slope, we know that the diagram indicates electron energies and the slope should be such that electrons drift “downhill” in the field when E points “ uphill” in the band diagram.

2.2.2- Equilibrium:

Definiton:

1. Equilibrium refers to a condition of no external excitation except for temperature and no net motion of charge I.e. a sample at cont temp, in the dark and no fields applied on other hand.

2. Steady state: Refers to a non-equilibrium condition in which all processes are constant and are balanced by opposing processes e.g. A sample with a constant current or a constant generation of EHP's balanced by recombination (i.e., all transients have expired!) any equilibrium is at steady state but not vice versa.

For a semiconductor at equilibrium no net current flows.

$$Jn(x)=0,\ Jp(x)=0,\ \varepsilon = -dv/dx$$

From 2.2a

$$q\mu_n n_o(x)\varepsilon(x) + qD_n dn_o(x)/d(x) = 0 \quad \text{-------------------- } 2.5$$

$$n_o = N_c e^{-(Ec-Ef)/kt}$$

$$dn_o/dx = -1/kt \ .\ N_c\,(dEc/dx - dE_f/dx)\, e^{-(Ec-Ef)/kt} = n_o\,(q/kt)(dv/dx)$$

Substitute and simplify:

Thus Eq 2.5 ➔ $D/\mu = kt/q = 0.026$ v for t=300K ------------------ 2.6 (Einstein relation)

Observation: Eq (2.6) allows calculating either “D” or “µ” from a measurement of the other. If applies to “n” or “p” type.

2.3- Continuity equations

Continuity equations:

$$\partial n/\partial t = g_n - r_n + 1/q\ .\ \partial J_n/\partial x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2.7a)$$

$$\partial p/\partial t = g_p - r_p + 1/q\ .\ \partial J_p/\partial x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ (2.7b)$$

g_n, g_p = electrons and holes generation rate (cm^3/ sec) Caused by external influence such as optical excitation.

r_n = electron recombination rate in p-type semiconductor under low-level injection (i.e. $\Delta p << p_o$ for majority carriers)

Then $r_n = n_p - n_{po}/\tau_n = n'/\tau_n$ …………………………………… (2.8a)

Where n_p = majority carrier density
N_{po} = thermal equilibrium minority density.
τ_n = minority carriers life time.

Similarly,
$r_p = p_n - p_{no}/\tau_p = p'/\tau_p$ …………………………………… (2.8b)

Note: If electrons and holes are generated and recombined in pairs with no trappings then $\tau_n = \tau_p$

Definition: Minority carrier Life time:

It is a decay constant ($Ae^{-t/\tau}$) and indicates the average time interval between the introduction and recombination of minority carriers.

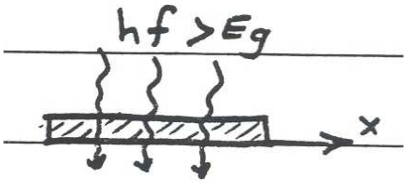
Fact: Continuity equations are derived from Maxwell's equation:

$\nabla .J = -\partial p/\partial t$ (i.e. conservation of charge)

Eq: $\partial j/\partial x = -\partial/\partial t(-q_n)+q(r_n-g_n) \Rightarrow \partial n/\partial t = g_n-r_n+(\frac{1}{q}\ \partial j/\partial x)$

2.4- Non-Equilibrium carrier concentrations:

Consider an extrinsic s/c (n-type) optically excited to generate EHP's by photons with sufficient energy ($hf \leq E_g$). This creates an equal number of conduction electrons and valence holes, similar to thermal excitation of electrons.



Total combinations are:

$n=n_0+n'$ and $p=p_0+p'$

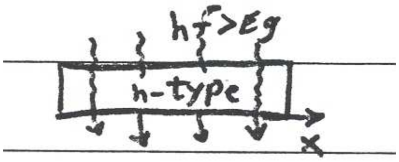
Example 1: Decay of Photo excited carriers:

Assume the illumination has been at steady state but at t=0, it is removed. EHPS are generated uniformly throughout the sample with a generation rate g. show the variation of p_n with time.

Solve: Prior to t=0(i.e. at $t=0^-$)

Boundary conditions (B.C) are :

$$\begin{cases} E = 0; \\ \partial p_n/\partial x = 0 \text{ Uniform generation;} \\ \partial p_n/\partial t = 0 \text{ Steady State} \end{cases}$$



Therefore, Cont. Eqn. => $\partial p_n/\partial t = g-(p_n - p_{no} / \tau_p)$ => $p_n = p_{no} + g\,\tau_p$ = const.

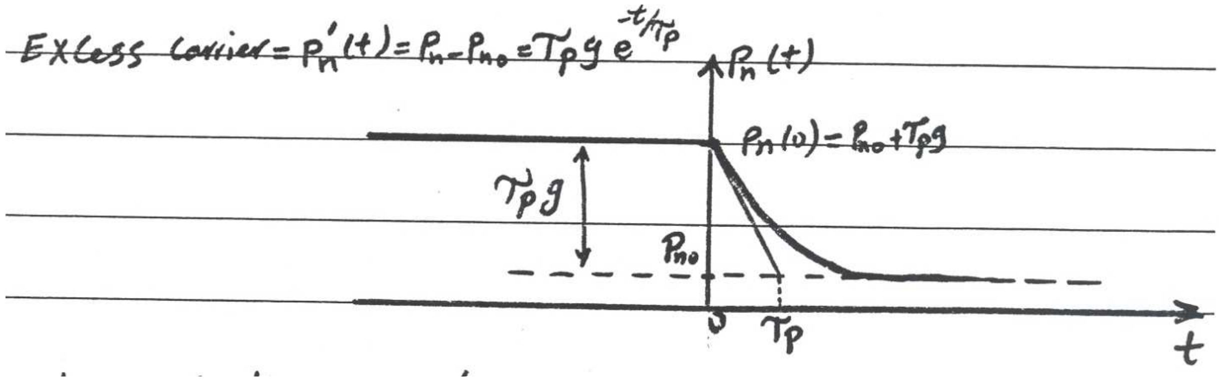
At $t=0^+$ Boundary conditions are:

$$\begin{cases} p_n(0)= p_{n0}+g\,\tau_p \\ P_n(t)/_{t\rightarrow\infty} = p_{n0} \end{cases}$$

Cont. Eqn. => $\partial p_n/\partial t = (p_n - p_{no} / \tau_p)$

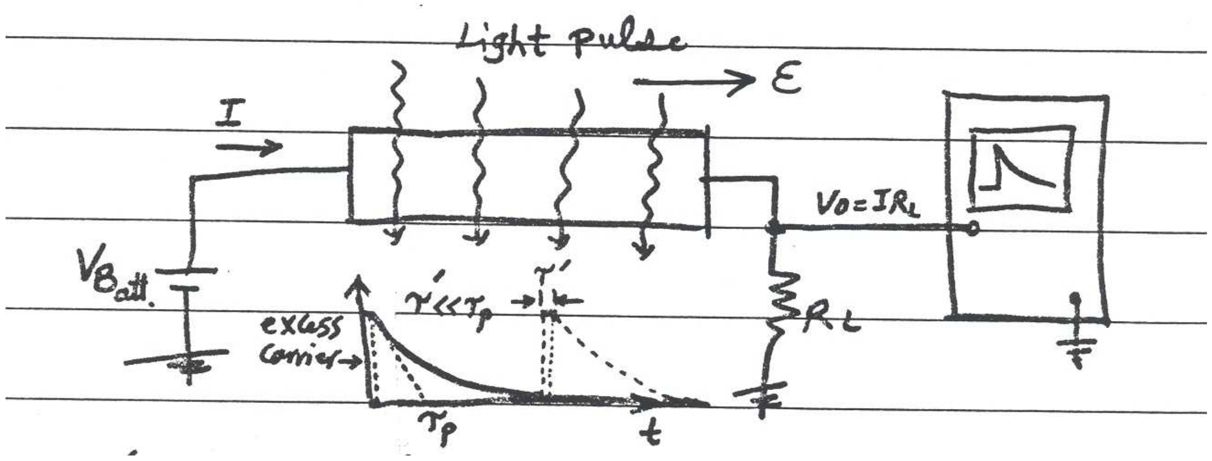
$P_n(t) = p_{n0} + \tau_p g e^{-t/}\,\tau_p$ (satisfies B.C.) (2.9)

Excess carrier = $P_n'(t) = p_n - p_{n0} = \tilde{I}_p g e^{-t/}\,\tilde{I}_p$



Note: According to Stevenson keys method, this can be used to measure minority carrier lifetime (τ_p)

A possible setup is as follows:



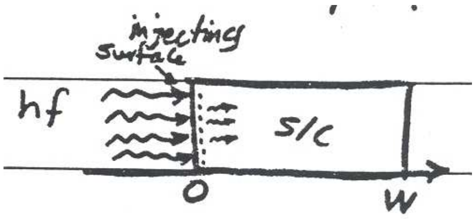
If a uniform light pulse is shined for a short time, the EHP generation causes an increase in δ=> increase in I. When the pulse is gone the decay of this conductivity is a measure of $\tau_{p.}$ The only condition is that the pulse width<< τ_p.

Example 2: Steady state injection from one side:

Consider the figure below where excess carriers are injected from one side so that EHPs are generated at the surface only. Find the minority carrier distribution as a function of X.

a) Let $w \rightarrow \infty$

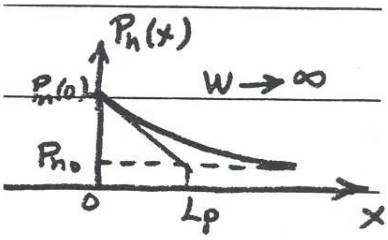
B.C $p_n(x=0)=\text{const.} = p_n(0)$



$p_n(x=\infty)= p_{n0}$

Continuity eqn: $\partial p_n/\partial t = -p_n - p_{no} / \tau_p - 1/q\ (\partial j_p/\partial x)$

Current density equation (Eq: 2.2b): $j_p = q\mu_p p\varepsilon - qD_p \partial p/\partial x$



➔ $-(p_n - p_{n0})/ \tau_p + D_p (\partial^2 p_n/\partial x^2) = 0$ (2.10)

➔ $p_n(x)= p_{n0}+[p_n(0)- p_{n0}]e^{-x/2p}$ (2.11)

Where $L_p= \sqrt{D_p \tau_p}$ is the diffusion length

$P'(X) = P_n(x)-P_{no} = [p_n(0)- p_{n0}]e^{-x/Lp}$ (2.12)

b) W is finite and it Extract all excess carriers at X=W

B.C $p_n(0)$ = const.

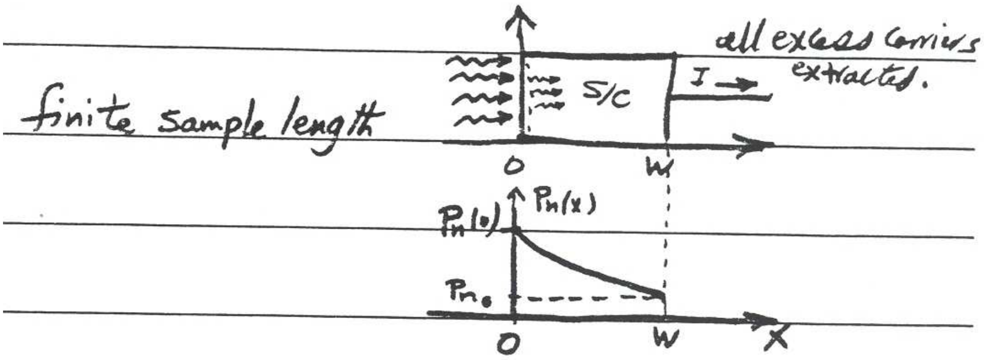
$P_n(w)= p_{no}$

Eq. (2.10) =>$p_n(x)=p_{no}+[p_n(0)-p_{no}] \left[\sinh(w-x/Lp)/\sinh(w/L_p) \right]$ (2.13)

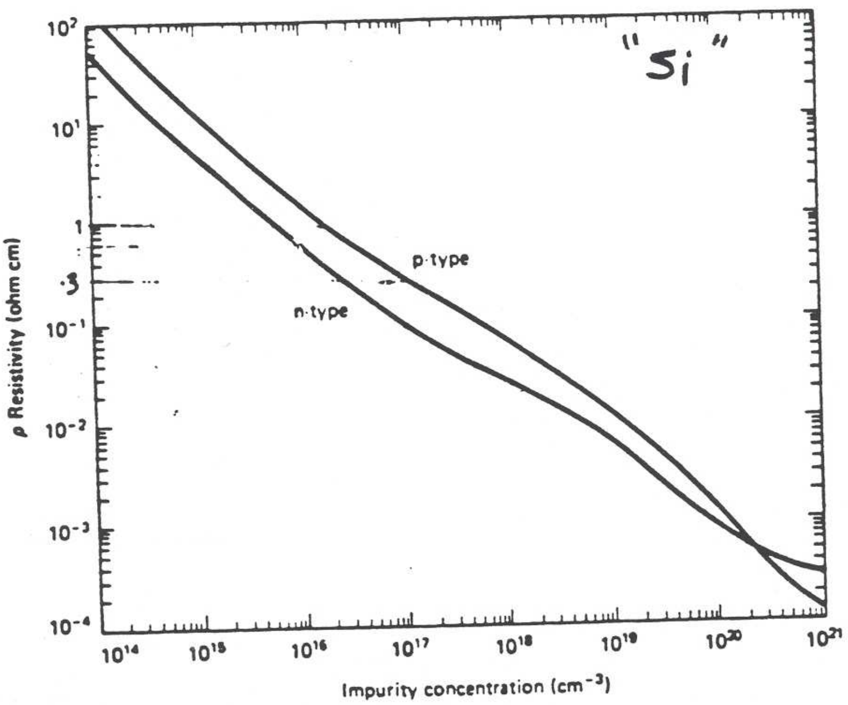
Current density extracted at X=W is

$J_p(w)=-qDp(\partial p/\partial x) = q[p_n(0)- p_{n0}]\ (D_p/L_p)[\sinh(w/L_p)]$ at x=w (2.14)

Note: Eq (2.14) can be shown to be related to the base current in an npn bipolar junction transistor.



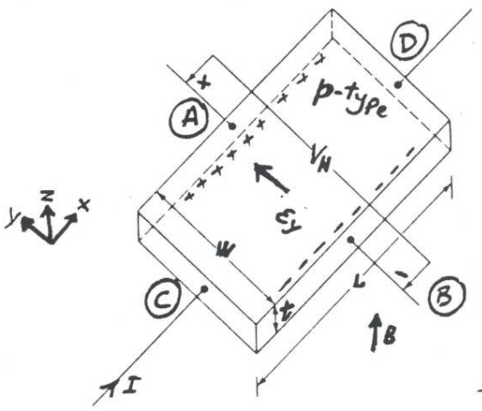
Silicon resistivity: (n type and p type)



2.5 The Hall Effect

The Hall Effect is based on the force exerted on a moving charge as it passes through a magnetic field. It is an important research for determining the carrier type(e^- or h^+),concentration (n,p) and mobility (μ_n , μ_p) for the majority carriers in the semiconductor.

Geometry for Max.
Hall effect →



Assume : p-type S/C ,
$\bar{B} = B_z \hat{z}$ (uniform)
I in $\hat{x}$ dir

$I = \dot{J} A = \sigma E_x A \Rightarrow$

$$I = (Wt)(qn\mu_n)\mathcal{E}_x \Rightarrow \mathcal{E}_x = I/Wtqn\mu_n \qquad (8.40)$$

$$F = q\bar{v} \times \bar{B} \Rightarrow \bar{F} = (+q v_x \hat{x}) \times (B_z \hat{z}) = -q v_x B_z \hat{y} = -q \mu_n \mathcal{E}_x B_z \hat{y} \qquad (8.41)$$

∴ Force is "$-\hat{y}$" dir. and causes "−" charge accumulate on the "−y" side & "+" charge on the "−y" side (i.e. e^- deficiency)

At steady state, an electric field "$\mathcal{E}_y$" counteracts the force due to $\bar{B}$.

$$+q\mathcal{E}_y = -q \mu_n \mathcal{E}_x B_z \xrightarrow{\text{use } (8.40)} \mathcal{E}_y = -\frac{IB_z}{Wtqn} \qquad (8.42)$$

and $$V_H = +\mathcal{E}_y W = \frac{IB_z}{tqn} = \text{induced Hall voltage} \qquad (8.43)$$

– The sign of this voltage determines the majority carrier type!

a. The magnitude of (V_H) determines the majority carrier concentration (n) for the s/c.

b. If a measurement of resistance (R) is made, ρ can be calculated :

$R = \rho L/wt \Rightarrow \rho = Rwt/L \Rightarrow (V_{CD}/I_x)/(L/wt)$ (8.44)

c. Finding ρ allows mobility to be found next as :

$\sigma = qn\mu_n \Rightarrow \mu_n = \sigma/qn \Rightarrow (1/\rho)/qn \Rightarrow 1/\rho qn$ (8.45)

Note 1:

If p-type material is used, h^+'s are deflected toward the same side (i.e."-Y") since h^+'s flow opposite to e^-'s. Therefore V_H is negative for p-type material but has the same form as eqn. (8.43).

Note 2:

Hall coefficient is defined to be:

$R_H = 1/qn$ (8.46)

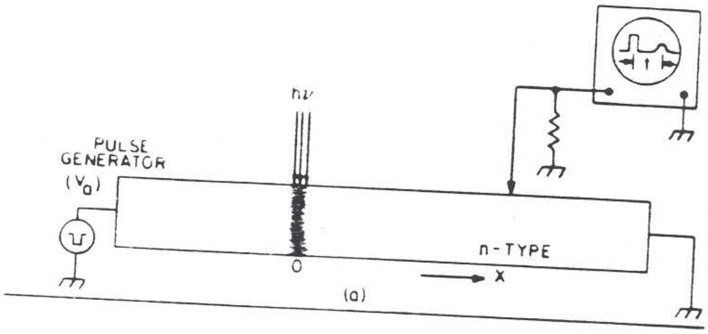
Therefore, eq 8.42 => $\varepsilon_y = R_H J_X B_z$ (8.47)

Eq 8.45 => $\mu_n = R_H / \rho$ (8.48)

& $n = 1/q R_H = (j^* B_z \quad)/q \varepsilon_y = (I_x/wt) B_z / q(V_{AB}/w) \Rightarrow n = I_x B_z / qtV_{AB}$ (8.49)

2.6 Haynes-Shockley Experiment

When localized light pulses generate excess carriers in a s/c (as shown below), the continuity eqn. after the pulse (g=0) is given by



Eq. (2.7a) gives

$\partial p/\partial t = g_p - r_p - 1/q(\partial j_p/\partial x)$ (2.15)

$$\begin{cases} r_p = p_n - p_{no} / \tau_p & (2.16) \\ j_p = q\mu_p \rho \varepsilon - qD_p \partial p/\partial x \Rightarrow \partial j_p/\partial x = q\mu_p \varepsilon \partial p/\partial x - qD_p \partial^2 p/\partial x^2 & \end{cases}$$

eq (2.15) => $\partial p/\partial t = - p_n - p_{no} / \tilde{I}_p - \mu_p \varepsilon \partial p_n/\partial x + D_p \partial^2 p_n / \partial x^2$ (2.17)

a) **Case 1:** (E = 0)

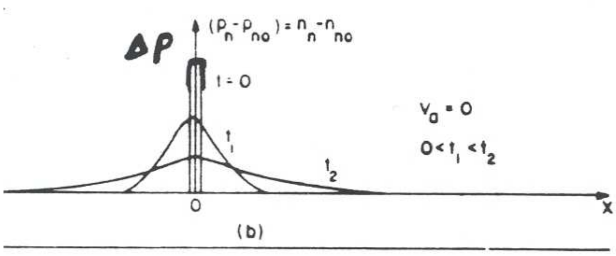
No E-Field is applied i.e. $\varepsilon = 0$

Soln. to Eqn. (2.17) is a “Gaussian Distribution”;

At $t=o^+$ pulse is removed and the excess carriers diffuse as :

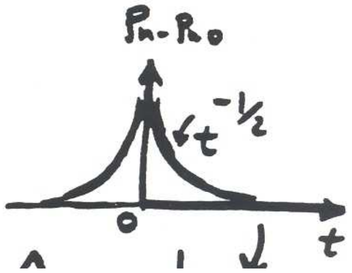
$P_n(x,t) = P_{no} + \Delta p/(2\sqrt{\prod D_p t})\ .\ b\ \ e^{-x2/4Dt-t/\tau p})$ (2.18)

Where Δp is excess holes at t=0(per unit area) over a small distance (δx)



Note: The exponential factor predicts the spread of pulse in the positive and negative X directions.

Check:



$t=o;\ P_n - P_{no} = \ \Delta p/\ \delta x$

$t=\infty;\ P_n - P_{no} = 0$

$x=0;\ P_n - P_{no} = \Delta p/(2\sqrt{\prod D_p t})$

$x=\infty;\ P_n - P_{no} = 0$

Case 2: ($\varepsilon \neq 0$)

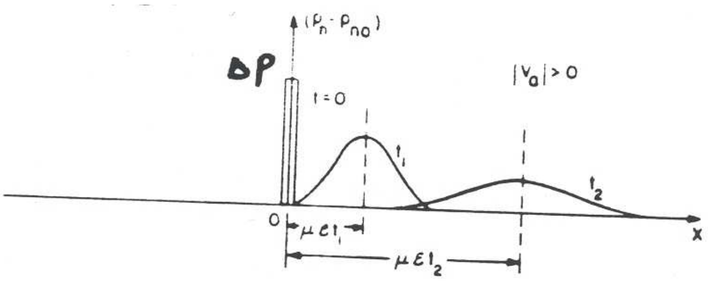
An electric- field (ε or E) is applied along the sample; the solution is the same as eq. (2.18) except “x” should be replaced with “$x- \mu_p \varepsilon t$”.

Thus the whole package of excess carriers moves towards the negative end of the sample with the drift velocity “$\mu_p \varepsilon$”.

At the same time, the carriers diffuse outward and recombine as in the field-free ($\varepsilon=0$) case.

$P_n(x,t)=P_{no}+\Delta p/(2\sqrt{\prod}D_pt)\ .\ e^{-x2/4Dt-t/\tau p})$

$X_o= \mu_p\varepsilon t.$



In this case, D_p can be calculated from a simple measurement as follows:

At $t=t_d$, $x=x_d+\Delta x/2$, ($x_d= \mu_p\varepsilon t_d$)

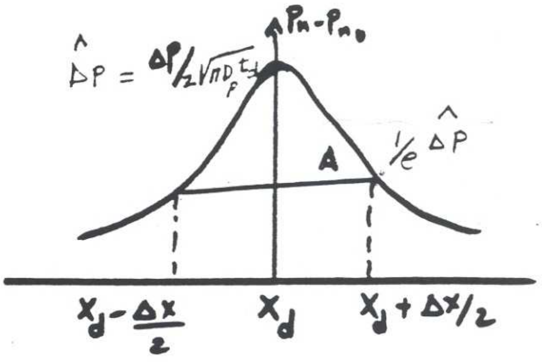
Use case 2 results (point A):

use case II results (pt. A):

$$e^{-1}\hat{\Delta p} = \hat{\Delta p}\, e^{-(\cancel{x_d} + \frac{\Delta x}{2} - \cancel{x_d})^2/4D_p t_d}$$

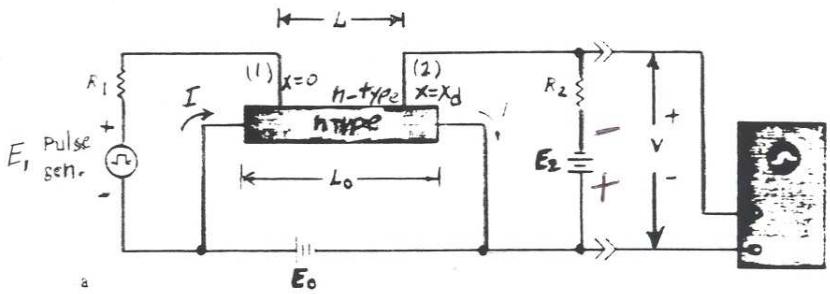
$$e^{-1}\hat{\Delta p} = \hat{\Delta p}\, e^{-(\frac{\Delta x}{2})^2/4D_p t_d}$$

$$\Rightarrow \boxed{D_p = \frac{(\Delta x)^2}{16 t_d}}$$

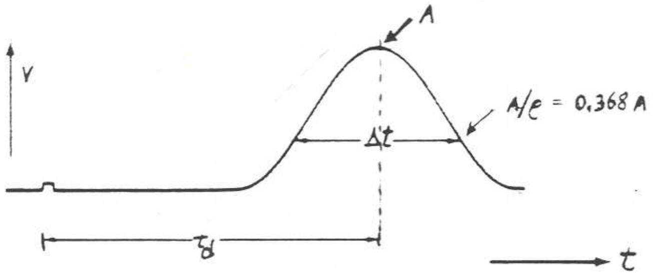


Since Δx can not be measured directly, an experimental setup (shown next) is used to display the pulse of $x=x_d$ on an oscilloscope as the carriers pass under a detector. We measure "Δt" and then relate it to Δx:

$\Delta x=V_d\Delta t = x_d\Delta t/t_d$



a



(b·

Let $L=x_d$= length between probes.

Therefore, $D_p=(\Delta x)^2/16t_d =(\Delta tL/ t_d)^2/16t_d$

$D_p = (\Delta tL)^2/16t^3_d$

Also μ_p can be found :

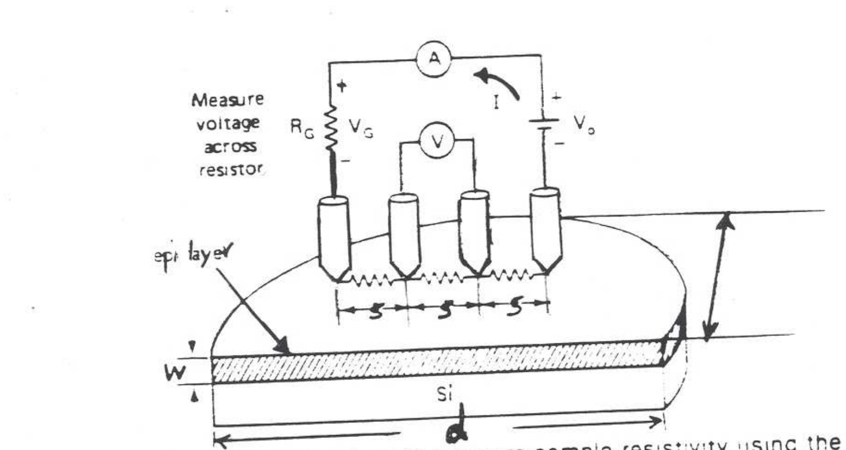
$V_d = L/t_d$

$\mu_p = V_d/\varepsilon \quad (\varepsilon=E_o/L_o)$

⇨ $\mu_p=L\ L_o/ E_o\ t_d$

2.7 “Four point” Probe method

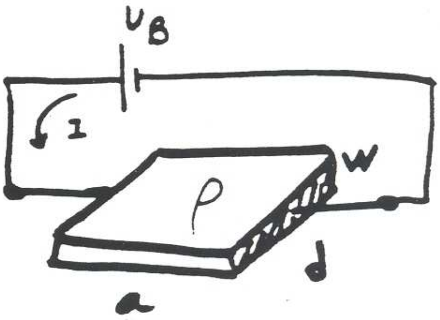
This is the most common method for measuring “ρ”, a small current from a const current source is passed through the outer two probes. For a thin wafer with thickness “w” much smaller than other dimensions (s or d), the sheet resistance can be calculated.



S= Probe Spacing

Figure 3-14. Experimental set-up to measure sample resistivity using the four-point probe technique.

Sheet Resistance concept:



R=ρa/wd

For a square,a=d and R=R_s

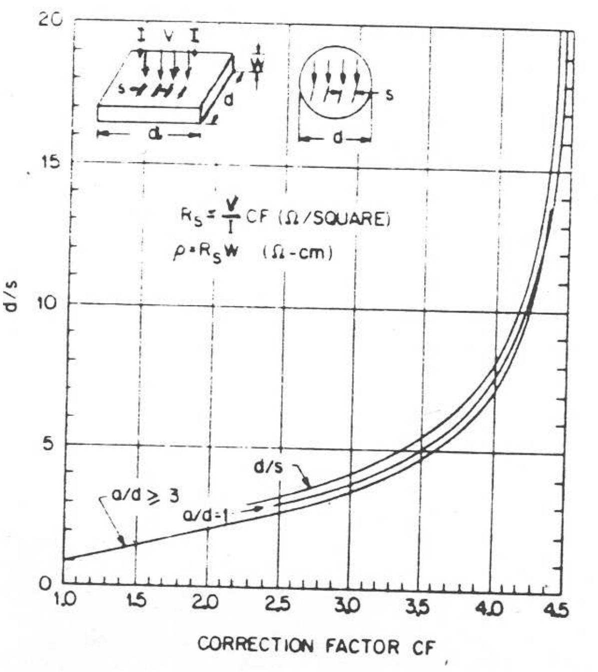
R_s=ρ/w (known by measurement)

R_s=(V/I) CF

Therefore, ρ =w R_s

Where CF=The collection factor is shown below.

Special case: In the limit when d>>s =>CF=π/ln2 = 4.54



<u>EXAMPLES</u>:

Ex 11 A)

A Sample of Si is doped with 10^{17} phosphorous atoms/cm^3. What would you expect to measure for its resistivity? What Hall voltage would you expect in a sample 100μm thick if I_r=1mA and

$B_z = 1\text{kg} = 10^{-5}\text{wb/cm}^2$.

Solution:

From Fig 3-2.3, the mobility is $700\text{cm}^2/(\text{V-s})$. Thus the conductivity is

$$\sigma = q\mu_n n_0 = (1.6 \times 10^{-19})(700)(10^{17}) = 11.2(\Omega\text{-cm})^{-1}$$

Since p_0 is negligible. The resistivity is

$$\rho = \sigma^{-1} = 0.0893\Omega\text{-cm}$$

The Hall coefficient is

$$R_H = -(qn_o)^{-1} = -62.5\text{cm}^3/\text{C}$$

From Eq (3-49), or we could use Eq (3.53). The hall voltage is

$V_{AB} = I_t B_z * R_H / t = (10^{-3}\text{A})(10^{-3}\text{wb/cm}^2)*(-62.5\ \text{cm}^2/\text{C})/10^{-2}$

$= -62.5\mu\text{v}$

Ex 11 B)

The p-type base region of the n-p-n bipolar silicon transistor has a width of 2.0×10^{-4}cm and is doped with 1.0×10^{15} acceptors/cm^3. Electrons are injected into this region from the emitter at x_t,producing a uniform gradient of electrons there with the electron concentration dropping to zero at the collector at x_c. If 2.0×10^{14}electrons/cm^3 are present at the emitter edge of the base region (x_E). (a) calculate the diffusion current density of electrons through this base region under steady state conditions (b) What electric field must be present in this base region to yield an electron drift current density exactly equal to the diffusion current density just calculated? (c) Determine the voltage drop across this base width corresponding to this field.

Solution : (a) From Appendix, $\mu_n = 1500\text{cm}^2/\text{v-sec}$, and from the Einstein relation , $D_n = (0.026\text{V}^{-1})(1500\ \text{cm}^2/\text{V-sec})$ or $D_n = 39\ \text{cm}^2/\text{sec}$. From Eqn (2.1)

$(J_n)_{\text{diffusion}} = q\, D_n\, dn/dx$

$= (1.6 \times 10^{-19}\text{C})(39\ \text{cm}^2/\text{sec})\ (2.0 \times 10^{14}/2.0 \times 10^{-4})\text{cm}^{-4}$

$= 6.2\ \text{A/cm}^2$

(b) from Eq (2.3)

$(J_n)_{\text{drift}} = qn\mu_n\varepsilon$ (use average electron density);

$6.2\ \text{A/cm}^2 = (1.6 \times 10^{-19}\text{C})(1.0 \times 10^{14}\text{cm}^{-1})(1500\ \text{cm}^2/\text{v-sec})(\varepsilon\ \text{V/cm})$

$\varepsilon = 2.6 \times 10^2$ v/cm.

(c) Voltage drop: $V = \varepsilon W = (2.6 \times 10^2)(2.0 \times 10^{-4}) = 5.2 \times 10^{-2}$ V.

Ex 12) A bar of n-type silicon at 300k contains 5×10^{15} donor impurity atoms /cm^3 and exhibits a minority hole life time of 1 µs. The sample is illuminated with light of wave length $\lambda = 8000$A. Which introduces 1.0×10^{14} excess electron-hole pairs/cm^3.

(a) Prove that this illumination will excite electron-hole pair in the sample.
(b) How long after the light is shut off will it take for the excess hole density to fall to 10% of its initial value.

Solution: E=hf; $f = C/\lambda$; E=Photon Energy.

(a) The photon energy for $\lambda = 8000 \times 10^{-8}$ cm is $\mathbf{E} = \frac{hc}{\lambda}$

$= (6.63 \times 10^{-14}\text{J-sec})(3 \times 10^{10}\text{cm/sec})/8000 \times 10^{-8}\text{cm}$
$= 2.5 \times 10^{-19}$J or $2.5 \times 10^{-19}\text{J}/1.6 \times 10^{-19}\text{J/ev} = 1.55$eV.

Since this photon energy exceeds the silicon gap width of 1.12eV. Electron-hole pairs will be excited across the gap.

(b) $\Delta p = \Delta p_o e^{-t/\tau}$ or $t = -r \ln(\Delta p/\Delta p_o)$
$= -(1.0 \times 10^{-6}\text{sec})\ln(0.10)$ and t=2.3µsec for the excess hole density to reduce to 10% of its original value.

Ex 13 A)

Let us assume a sample of GaAs is doped with 10^{15} acceptors/cm^3. The intrinsic carrier concentrations of GaAs is app. 10^6cm^{-3}; Thus the minority electron concentration is $n_o = n_i^2/p_o = 10^{-3}$cm^{-3}. Certainly the approximation $p_o >> n_o$ is valid in this case. Now if 10^{14} EHP/cm^3 are created at t=0, we can calculate the decay of these carriers in time. The approximation $\delta n << p_o$ is reasonable, as fig 4-7 indicates. The fig shows the decay in time of the excess populations for a carrier recombination life time of $\tau_n = \tau_p = 10^{-8}$s.

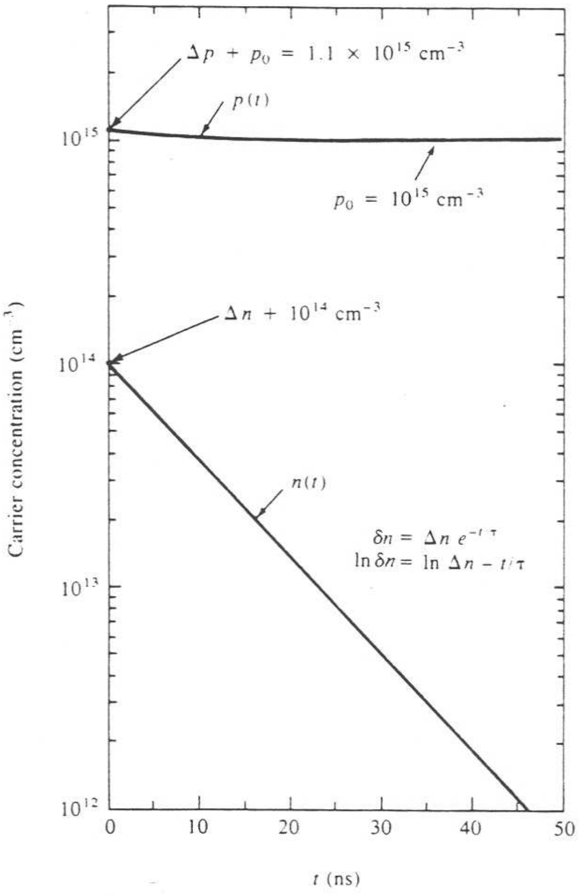


Figure 4-7
Decay of excess electrons and holes by recombination, for $\Delta n = \Delta p = 0.1p_0$, with n_0 negligible, and $\tau = 10$ ns (Example 4-2). The exponential decay of $\delta n(t)$ is linear on this semi-logarithmic graph.

There is a large percentage change in the minority carrier electron concentration in this example and a small percentage change in the majority hole concentration. Basically, the approximations of extrinsic material and low-level injection allow us to represent n(t) in Eq (2.7) by the excess concentration $\delta n(t)$ and p(t) by the equilibrium value p_o.

Note: $n(+) = n_o + \delta_n(t)$

Therefore, $\delta_n(t) = n'(t)$ excess carrier.